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NUMERICAL INVESTIGATION OF DAMPED PARALLEL RLC CIRCUIT: ANALYSIS AND INSIGHTS

[1]Abdul Saboor Khan*, [2]Salat Ahmed Khan, [1]Muhammad Wali Khan, [3]Ali Riaz, [1]Adeel Ahmed, [1]Muhammad Aqeel Gul
[1]The Faculty of Engineering, Science and Technology Iqra University Karachi Pakistan
[2]Associate Professor, Faculty of Mathematics, Education Department, Government of Sindh Pakistan,
[3]MS Scholar, Department of Mathematics, NED University of Engineering and Technology Karachi, Sindh
Emails: abdul.saboor@iqra.edu.pk, ahmadsalaatkhan@gmail.com, muhammad.wali@iqra.edu.pk, riaz.pg3200152@cloud.neduet.edu.pk, adeel.ahmed01@iqra.edu.pk, muhammad.aqeel@iqra.edu.pk

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Corresponding Author: *
Abdul Saboor Khan*

Abstract
This research addresses the analysis of damped parallel RLC circuits, consisting of resistors (R), inductors (L), and capacitors (C), which are fundamental components in modeling and understanding various electrical systems. The damping effects in these circuits significantly influence their transient behavior, stability, and overall performance. By employing the system of differential equations derived from Kirchhoff's voltage and current laws, this study explores the dynamic properties and responses of these circuits under damping conditions. The investigation utilizes advanced numerical methods—Euler Method, RK3 Method, and RK5 Method—to approximate solutions for voltage and inductor current in damped parallel RLC circuits. These methods are critically evaluated based on their computational efficiency and accuracy. The Euler Method, known for its simplicity, is suitable for quick estimations but lacks precision compared to higher-order methods. The RK3 Method provides a balance between accuracy and computational effort, making it effective for practical applications. The RK5 Method, a fifth-order technique, stands out for its exceptional accuracy, proving indispensable in scenarios requiring high precision. This study offers valuable insights into the impact of damping on the transient responses and stability of parallel RLC circuits. Numerical simulations reveal how damping alters the circuit's performance, shedding light on optimization strategies and design considerations. Through detailed graphical presentations, the research demonstrates the comparative strengths of these numerical methods, highlighting their applicability to solving complex circuit equations. By providing a comprehensive analysis of damping effects and numerical methods, this work contributes to a deeper understanding of the behavior of damped parallel RLC circuits, offering guidance for efficient and accurate circuit analysis in engineering and applied mathematics.

Introduction

RLC circuits are made up of resistors (R), inductors (L), and capacitors (C) which are very basic parts of electrical engineering and circuit design. They are used everywhere from communication systems that require filtering signals to tuning circuits in radio transmitters and receivers. The behavior of RLC circuits can be analyzed in both the time and frequency domains, making them essential for understanding transient responses and steady-state oscillations. Initially, the stress of the study in literature was on first-order circuits which involve either an inductor or capacitor as a reactive element and hence form a first-order differential equation. With time, the emphasis was shifted to the most common and wide applications of 2nd order RLC circuits that include both inductor and capacitor as reactive elements [1]. The higher-order RLC circuits are more intricate with complex configurations of multiple reactive inductors and capacitors and with non-standard configurations. Due to its complexity, its applications and usage are quite limited.

Second-order RLC circuits are demanding in various advanced electronics applications because of their appropriate handling of multifaceted frequency-dependent behaviors and transient responses. The most common arrangement of RLC circuit is series RLC in which the component is arranged in series and the total voltage of the system is obtained by adding the voltage across each component [1]. On the other side, the parallel RLC circuits have arrangements of components in parallel in the same two points. Parallel RLC circuits are important components in electronic systems and hence very interesting for providing insight into the study of electrical engineering [1] [2] . These RLC parallel circuits have dynamic behavior executed by the components such as resistors, inductors and capacitors that require complex analysis for design and performance. The basic understanding of these circuits has evolved continuously from the molding influence of Kirchhoff and Maxwell, to modern numerical methods applied in detail [3] [4]. Specifically, this research focuses on dynamic properties influenced by damping effects and presents a numerical analysis of damped parallel RLC circuits [5] . The scope ranges from sophisticated numerical techniques to general conclusions with a generic influence. However, the study is expected to identify and overcome some limitations (computational bounds or simplified circuit model). Parallel RLC circuits are indispensable for electronic systems, they may be difficult to analyze, especially when damping effects are included [6] . Due to the effects of damping on transient responses and circuit stability, constructing reliable as well as responsive circuits requires significant investigation. A substantial body of literature exists, yielding powerful insights in the same; some gaps and constraints continue to exist although most significant works have been conducted but demands further detailed numerical analyses [7].

RLC circuits are the essential circuit elements for all electronic systems and can be vital part of several power supplies, amplifiers, filters, and other applications. The attributes of the circuits are evaluated by the interchange of arrangement of resistors (R), inductors (L) and capacitors in parallel or in series. In assembling electronic equipment, it is crucial to apprehend the dynamics respond of parallel RLC circuits. The performance of electrical circuits are well established by classic circuit theories from the work done by Maxwell and Kirchhoff. However, with growing complex electronic systems, the need of the time is to analyze and predict the behavior of these electrical circuits by advanced methods is expanding [8] [9].

It is essential to study the damping analysis in parallel RLC circuits which helps us understand transient response and stability of electronic systems. The process of energy being passed from one part to another in a circuit is known as damping, it can be helpful when predicting how the system will behave under certain conditions. Systems such as power electronics and communication systems must have their damping tuned. Derived from fundamental microcontroller mode-coupling equations, the critical damping phenomenon (underdamping and over damping) is crucial to the implementation of resonant controllers in power electronics. A study on damping events must be carried out to create circuits that meet certain performance standards. [10] [11].

This study's scope includes an entire numerical examination of damped parallel RLC circuits bearing in mind different setups and parameter values. Well-established and deep-rooted numerical methods are used to check the performance of the circuit simulation. Its authorities identify certain limitations, limited to the assumption of superlative elements somehow, certain lapse parasitic apart, and the calculations linked with intrinsic numerical techniques. Author prefer to use the authentic tool for data analysis that removes ambiguities upon certain limitations. [12] [13].

Consideration of the behavior of parallel RLC circuits is critical to investigating dampening effects. A summary of the theoretical reinforcements is given in this part, with special consideration to the jobs of resistors (R), inductors (L), and capacitors (C) in these circuits. It researches deeper on the idea of damping in electrical circuits, prominence circumstances where damping is critical, overdamping, and underdamping. Moreover, covered are theoretical reinforcements of numerical techniques like the RK3 and RK5 methods. The performance of resistors, inductors, and capacitors, different types of damping, their reputation in parallel RLC circuits, and numerical methods for circuit examination, such as RK3 Methods, RK5 Methods, and both time-lapse and frequency-domain examines, are all covered in detail in this theoretical outline section. This extended theoretical outline offers a profounder study of the behavior of parallel RLC circuits, damping effects, and investigative techniques operating numerical approaches.

Three essential mechanisms make-up a basic class of electrical circuits known as parallel RLC circuits: resistors (R), inductors (L), and capacitors (C). Since the parts of these circuits are linked in parallel, numerous formations that affect the circuit's overall performance are possible. The circuit's response to various input signals and circumstances is determined by the contact of the resistance, inductance, and capacitance. These basic ideas oversee the study of parallel RLC circuits, in which an electric field is kept by capacitors (C), a magnetic field is stored by inductors (L), and an impeded current flow is caused by resistors (R). Multifaceted circuit responses are the significance of these components' dynamic interaction, demanding the use of sophisticated analytical and numerical methods for precise prediction and investigation. [14] [15].

For efficient design and operation, real-world applications like power electronics and communication systems need a sophisticated grasp of damping effects. [16] [17]

The circuit response oscillates as a result of underdamping before it reaches the steady state. Its quicker settling time is its defining feature, yet oscillating behavior is a disadvantage. [18] [19]. A crucial component of parallel RLC circuits, damping affects the circuits' overall performance, transient responsiveness, and stability. To optimize the circuit's behavior in various applications, the damping ratio must be reached. It is essential to comprehend the relevance of damping kinds while developing circuits that meet certain criteria. [20] [21]. Time Domain Analysis (TDA) methods can be used to simulate the transient response of circuits and this way, circuits can be explained how they behave over time [22] [23]. One of them is using Fast Fourier Transform (FFT) etc. and frequency domain analysis to characterize the behavior of the circuit under different component frequencies.

The Runge-Kutta method is one of the most popular numerical methods for solving systems of ordinary differential equations. It is an iterative process. At each step, a weighted average of derivative values is computed to estimate the solution over discrete time steps. Runge-Kutta methods are used widely due to their higher order precision than many other techniques such as Euler's method. The most widely used is the fourth-order Runge-Kutta (RK4) method. The focus of RK approaches are on discretization in time for ODEs or sets of ODEs; in contrast to RK Method whereby the whole domain has to be discretized. These techniques can be applied to a variety of ordinary differential equation problems, including the simulation of chemical kinetics and mechanical systems. [24] [25] [26].

1.2 Research Gap

This section expands the critical analysis, pointing out the deficiencies and gaps that exist in the literature. The computational challenges and unknown areas of damped parallel RLC circuits are stated, and future research is pointed out. The identification of these gaps prepares the reader for the novelty and importance of the current study.

The available literature on damped parallel RLC circuits identifies specific limitations and challenges, such as the lack of comprehensive comparative studies on numerical methods for analyzing transient responses and stability, and the computational inefficiencies in handling complex circuit dynamics. These gaps highlight opportunities for future research to develop more robust numerical techniques, explore advanced damping characteristics, and enhance the understanding of circuit behavior under varying conditions. This study aims to address these less-explored areas, offering new insights into computational approaches and practical applications.

2. Numerical Methodologies

This section outlines the numerical methods adopted for analyzing damped parallel RLC circuits. It justifies the chosen methods, provides a detailed description of the simulation setup (parameters, initial conditions, and boundary conditions), and discusses the validation process. Comparisons with analytical solutions and experimental verifications, if available, are crucial to establishing the credibility of the chosen methodology.

This methodology section outlines the numerical methods chosen for the analysis, justifies their selection, describes the simulation setup including parameters and conditions, and details the validation process involving comparisons with analytical solutions and, if applicable, experimental verification.

A resistor, an inductor, and a capacitor are the three electrical components that make up an RLC circuit. The circuit is of the second order. These circuits are the most often used because they may be used to build radio or audio receiver tuners and oscillators [27] . The parallel RLC circuit is represented by the electrical components resistor (R), inductor (L), and capacitor (Ü) in Fig: A1, which are linked in parallel with the D.C. source. To complete the analytical solution portion of a transient study, a circuit equation is created [28]. A typical electrical circuit may have hundreds of parts. It is therefore nearly hard to find any analytical solution in such RLC circuits. Numerical techniques could provide significant alleviation for the system's solution in such a situation [29]

Python compiler is being used as computational tool for this research along with the different analytical methods through which research is being conducted.

Parallel RLC Circuit

In a parallel RLC circuit, the resistor (R), inductor (L), and capacitor (C) are connected in parallel. The governing differential equation for the circuit can be derived from Kirchhoff's Current Law (KCL), which states that the total current I(t) is the sum of the currents through each component:

$$I(t) = I_R(t) + I_L(t) + I_C(t) \quad (3.1)$$

where $I(t)$ is the total current source, and $I_R(t)$, $I_L(t)$, and $I_C(t)$ are the currents through the resistor, inductor, and capacitor, respectively.

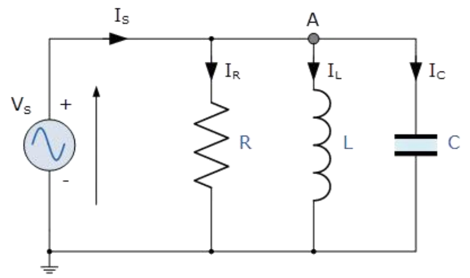

Fig: A1

1. **Current through the Resistor $I_R(t)$:**

$$I_R(t) = \frac{V(t)}{R}$$

2. **Current through the Inductor $I_L(t)$:**

$$V(t) = L\frac{dI_L(t)}{dt}$$

$$\frac{dI_L(t)}{dt} = \frac{V(t)}{L}$$

3. **Current through the Capacitor IC(t):**

$$I_C(t) = C\frac{dV(t)}{dt}$$

Combining these currents, we have:

$$I(t) = \frac{V(t)}{R} + C\frac{dV(t)}{dt} + \frac{1}{L}\int V(t)\,dt \quad (3.2)$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 24 \quad (3.3)$$

Initial Conditions and Parameters

- $V(0) = 6$ (Initial voltage across the capacitor)
- $I(0) = 0$ (Initial current through the inductor)
- $\frac{dV(0)}{dt} = 12$ (Initial rate of change of voltage)
- $R = 1$ (Resistance in ohms)
- $L = 1$ (Inductance in henries)
- $C = 0.5$ (Capacitance in farads)
- $\Delta t = 0.1$ s (time step)

After the switch is closed, the transient characteristics of the parallel RLC circuit are examined. Several iterative techniques can be used to solve equation (3.2).

The damping factor determines how much a system's oscillation progressively diminishes over time (t). The damping factor ב determines the transient responsiveness. The damping factor in a parallel RLC circuit is determined by

ב $= \frac{\alpha}{\omega_0}$ Where, $\alpha = \frac{1}{2RC}$ (damping coefficient) and $\omega_0 = \frac{1}{\sqrt{LC}}$(resonant frequency).

Then, ב $= \frac{1}{2R}\sqrt{\frac{L}{C}}$ (3.4)

The system is (i) overdamped when ב > 1, (ii) critically damped when ב $= 1$, and (iii) underdamped when ב < 1 [48].

To solve initial value problems (IVP) for ordinary differential equations (ODE), we introduce the explicit Euler approach, the third-order Ruge-Kutta method (RK3), and the 5th order Runge-Kutta method(RK-5) in this section [30] [31] [32].

The forward Euler formula is for a first-order differential equation of the form:

$$\frac{dy}{dt} = f(t, y) \quad (3.5)$$

with an initial condition $y(t_0) = y_0$

$$t_{n+1} = t_n + h$$

$$y_{n+1} = y_n + h \cdot f(t_n, y_n) \quad (3.6)$$

The RK3 formula is

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 4k_2 + k_3) \quad (3.7)$$

Where

$$k_1 = f(t_n, y_n)$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f(t_n + h, y_n + hk_1 + 2hk_2)$$

The RK5 formula

$$t_{n+1} = t_n + h \quad (3.8)$$

$$y_{n+1} = y_n + \frac{h}{90}(7k_1 + 32k_2 + 12k_3 + 32k_4 + 7k_5)$$

where $k_i$are the intermediate stages calculated as follows:

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{5}, y_n + \frac{h}{5}k_1)$$

$$k_3 = f(t_n + \frac{3h}{10}, y_n + \frac{3h}{40}k_1 + \frac{9h}{40}k_2)$$

$$k_4 = f(t_n + \frac{4h}{5}, y_n + \frac{44h}{45}k_1 - \frac{56h}{15}k_2 + \frac{32h}{9}k_3)$$

$$k_5 = f(t_n + h, y_n - \frac{16h}{135}k_1 + \frac{4h}{9}k_2 - \frac{4h}{5}k_3 + \frac{16h}{135}k_4)$$

It can extend the above discussed iterative methods for the solution of higher-order IVP for ODE and system of IVP for ODEs [33].

Numerical formulation of parallel RLC circuit

For the parallel RLC circuit:

$$\frac{dV}{dt} = \frac{I(t) - V(t)_R - I_L(t)}{C} \quad (3.9)$$

$$\frac{dI_L}{dt} = \frac{V(t)}{L} \quad (3.10)$$

2.1 Euler's method for RLC circuit

The Euler method is a simple and widely used numerical technique for solving ordinary differential equations (ODEs). It is particularly useful for simulating the dynamic behavior of electrical circuits, such as the parallel RLC circuit. This report demonstrates the application of the Euler method to solve the first 10 steps of a parallel RLC circuit, given specific initial conditions and parameters.

Using the Euler method, the updates for voltage V and inductor current I_L at each step n are:

$$V_{n+1} = V_n + \Delta t \cdot \frac{dV}{dt}|_{tn}$$

$$I_{L_{n+1}} = I_{L_n} + \Delta t \cdot \frac{dI_L}{dt}|_{tn}$$

Step-by-Step Calculation

Initial values:

$V_0 = 6, I_{L0} = 0$ and $\frac{dV}{dt}|_{t=0} =- 24$

First step:

$$\frac{dV}{dt}|_{t=0} = \frac{I(0) - \frac{V(0)}{R} - I_{L0}}{C} = \frac{0 - \frac{6}{1} - 0}{0.25} =- 24$$

$$\frac{dI_L}{dt}|_{t=0} = \frac{V(0)}{L} = \frac{6}{1} = 6$$

$$V_1 = V_0 + \Delta t \cdot (-24) = 6 + 0.01 \cdot (-24) = 5.76$$

$$I_{L1} = I_{L0} + \Delta t \cdot 6 = 0 + 0.01 \cdot 6 = 0.06$$

2.2 RK-3 method for RLC circuit

The RK3 method uses the following steps to update the voltage V and inductor current IL at each time step n:

Update equations for RK3 method:

$$k_1 = f(t_n, y_n)$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f(t_n + h, y_n + hk_1 + 2hk_2)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 4k_2 + k_3)$$

Step-by-Step Calculation

Initial values:

$V_0 = 6, I_{L0} = 0$

First step:

$$\frac{dV}{dt}|_{t=0} = \frac{I(0) - \frac{V(0)}{R} - I_{L0}}{C} = \frac{0 - \frac{6}{1} - 0}{0.25} =- 24$$

$$\frac{dI_L}{dt}|_{t=0} = \frac{V(0)}{L} = \frac{6}{1} = 6$$

$$k_V 1 = \frac{I_{L0} - \frac{V_0}{R}}{C} = \frac{0 - \frac{6}{1}}{0.25} =- 24$$

$$k1_{IL} = \frac{V_0}{L} = \frac{6}{1} = 6$$

$$k2_V = \frac{\left(I_{L0} + \frac{h}{2} k1_{IL}\right)}{C} - \frac{\left(V_0 + \frac{h}{2}k1_V\right)}{R} = \frac{(0 + 0.01 \cdot 3) - (\frac{6 - 0.01 \cdot 12}{1})}{0.25} =- 22.44$$

$$k2_{IL} = \frac{V1 + \frac{h}{2} k1_V}{L} = \frac{5.7654 - 0.01 \cdot 11.41}{1} = 5.6593$$

$$k_{3,V} = \frac{(I_{L1} - hk1_{IL} + 2hk2_{IL}) - \frac{(V\_1 - hk1\_V + 2hk2\_V)}{R}}{C}$$

$$= \frac{(0.0605 - 0.01 \cdot 5.7654 + 0.02 \cdot 5.6593) - \frac{(5.7654 - 0.01 \cdot 11.41 + 0.02 \cdot - 21.263)}{1}}{0.25}$$

$$= - 20.251$$

$$k_{3,IL} = \frac{V_1 - hk1_V + 2hk2_V}{L} = \frac{5.7654 - 0.01 \cdot 11.41 + 0.02 \cdot - 21.263}{1} = 5.4463$$

$$V_2 = V_1 + \frac{h}{6} (k1_V + 4k2_V + k3_V)$$

$$= 5.7654 + \frac{0.01}{6} (- 22.8202 + 4 \cdot - 21.263 + - 20.251) = 5.5344$$

$$I_{L2} = I_{L1} + \frac{h}{6} (k1_{IL} + 4k2_{IL} + k3_{IL})$$

$$= 0.0605 + \frac{0.01}{6} (5.7654 + 4 \cdot 5.6593 + 5.4463) = 0.1184$$

2.3 RK-5 method for RLC circuit

The RK5 method involves computing several intermediate slopes. We denote:

$f_1(t, x_1, x_2) = x_2$

$f_2(t, x_1, x_2) = 24 - 4x_2 - 4x_1$

Given the time step $\Delta t = 0.1$s, let's compute the RK5 coefficients.

Step-by-Step Calculation

Initial values:

- $x_1(0) = v(0) = 6$
- $x_2(0) = \frac{dv(0)}{dt} = 12$

Compute intermediate values:

Compute $\boldsymbol{k_{1,i}}$:

$k_{1,1} = \Delta t \cdot f_1(t, x_1, x_2) = 0.1 \cdot x_2 = 0.1 \cdot 12 = 1.2$

$k_{1,2} = \Delta t \cdot f_2(t, x_1, x_2) = 0.1 \cdot (24 - 4x2 - 4x1) = 0.1 \cdot (24 - 4 \cdot 12 - 4 \cdot 6) = 0.1 \cdot (24 - 48 - 24)$

$= 0.1 \cdot (- 48) = - 4.8$

Compute $\boldsymbol{k_{2,i}}$

$$k_{2,1} = \Delta t \cdot f_1(t + \frac{\Delta t}{4}, x_1 + \frac{k_{1,1}}{4}, x_2 + \frac{k_{1,2}}{4})$$

Substitute

$t + \frac{\Delta t}{4} = 0 + 0.025 = 0.025,\ x_1 + 41.2 = 6 + 0.3 = 6.3,\ and\ x_2 + \frac{-4.8}{4} = 12 - 1.2 = 10.8$:

$k_{2,1} = 0.1 \cdot 10.8 = 1.08$

$k_{2,2} = 0.1 \cdot (24 - 4 \cdot 10.8 - 4 \cdot 6.3) = 0.1 \cdot (24 - 43.2 - 25.2) = 0.1 \cdot (- 44.4) = - 4.44$

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$$k_{6,2} = 0.1 \cdot (24 - 4 \cdot 11.76585 - 4 \cdot 6.5371) = 0.1 \cdot (24 - 47.0634 - 26.1484) = 0.1 \cdot (-49.2118) = -4.92118$$

Update the Values

Using the weighted average method of RK5:

$$x_{1,new} = x_1 + \frac{1}{90}(7k_{1,1} + 32k_{3,1} + 12k_{4,1} + 32k_{5,1} + 7k_{6,1})$$

$$x_{2,new} = x_2 + \frac{1}{90}(7k_{1,2} + 32k_{3,2} + 12k_{4,2} + 32k_{5,2} + 7k_{6,2})$$

Substitute the k_i values:

$$x_{1,new} = 6 + \frac{1}{90}(7 \cdot 1.2 + 32 \cdot 1.0845 + 12 \cdot 1.1974 + 32 \cdot 1.16097 + 7 \cdot 1.176585)$$

$$x_{2,new} = 12 + \frac{1}{90}(7 \cdot (-4.8) + 32 \cdot (-4.44) + 12 \cdot (-4.8846) + 32 \cdot (-4.68956) + 7 \cdot (-4.92118))$$

Finally, compute these sums:

$$x_{1,new} = 6 + \frac{1}{90}(8.4 + 34.704 + 14.3688 + 37.95104 + 8.235095) = 6 + \frac{1}{90}(103.658935) = 6 + 1.15065 = 7.15065$$

$$x_{2,new} = 12 + \frac{1}{90}(-33.6 - 142.08 - 58.6152 - 150.04 - 34.44826) = 12 + \frac{1}{90}(-418.78346) = 12 - 4.652 = 7.348$$

Thus, the updated values at $t = 0.1$ s are approximately:

$v(0.1) \approx 7.15065$

$\frac{dv(0.1)}{dt} \approx 7.348$

1. Numerical Experiments and its set up

In the next section, the study will show that by numerical approaches defined above one can compare precise solutions with those of RLC circuits and use an iterative method to investigate into what properties a damping factor fall.

The simulation uses a time step size of 0.01 seconds. Simulation results from top, middle and bottom overlaid with the numerical solution for a parallel RLC circuit (in all its overdamped, critically damped and underdamped realizations) together with exact solutions are shown. Computed values of numerical solutions are at a specific time. The computed points taken around the whole spatial domain from all numerical methods are then compared.

.1 Results

It is simple to simulate the behavior of a parallel RLC circuit by using an Euler method. By successively updating the voltage and current numbers, discretizing time finally we are able to have a dynamic response across every circuit. The results should ideally observe the voltage to decrease exponentially and that of inductor's current respects it analogously.

In the solution of ordinary differential equations (ODEs), the third order Runge-Kutta (RK3) method is a numerical technique higher in accuracy compared to that Euler's algorithm. This paper describes the RK3 method to solve first 10 stages for a parallel RLC circuit using some initial conditions of settings.

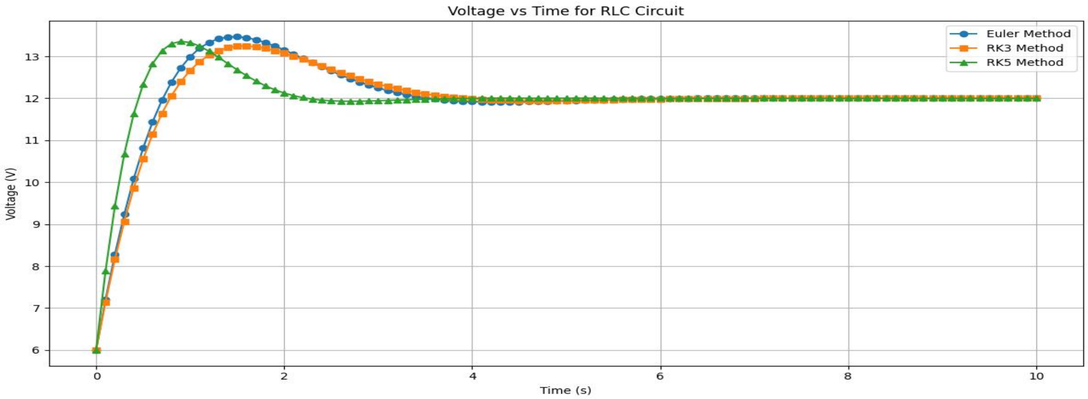

Fig: 1 It shows the resistance effect on Voltage for over-damped (R=1, C=0.5, L=1)

Table 1: Comparison results of the numerical methods (Exact, Euler, RK3 and RK5) over damped R=1, C=0.5, L=1 for the voltage of the circuit.

Time (s)	Exact Voltage (V)	Euler Voltage (V)	RK3 Voltage (V)	RK5 Voltage (V)
0.0	6.000	6.000	6.000	6.000
0.1	6.960	6.912	6.958	6.960
0.2	7.918	7.833	7.912	7.919
0.3	8.887	8.761	8.876	8.887
0.4	9.868	9.689	9.856	9.869
0.5	10.864	10.621	10.849	10.864
0.6	11.875	11.564	11.857	11.875
0.7	12.905	12.532	12.870	12.905
0.8	13.954	13.527	13.908	13.954
0.9	15.023	14.530	14.973	15.023
1.0	16.114	15.563	16.057	16.114

Table 2: Comparison results of the three numerical methods (Exact, Euler, RK3 and RK5) over damped R=1, C=0.5, L=1 for the Current

Time (s)	Exact Current (A)	Euler Current (A)	RK3 Current (A)	RK5 Current (A)
0.0	12.000	12.000	12.000	12.000
0.1	10.980	10.988	10.979	10.980
0.2	10.045	10.092	10.060	10.048
0.3	9.170	9.207	9.178	9.170
0.4	8.357	8.402	8.368	8.356
0.5	7.601	7.654	7.609	7.600
0.6	6.900	6.969	6.908	6.900
0.7	6.251	6.338	6.253	6.251
0.8	5.652	5.669	5.652	5.652
0.9	5.100	5.105	5.100	5.100
1.0	4.592	4.588	4.592	4.592

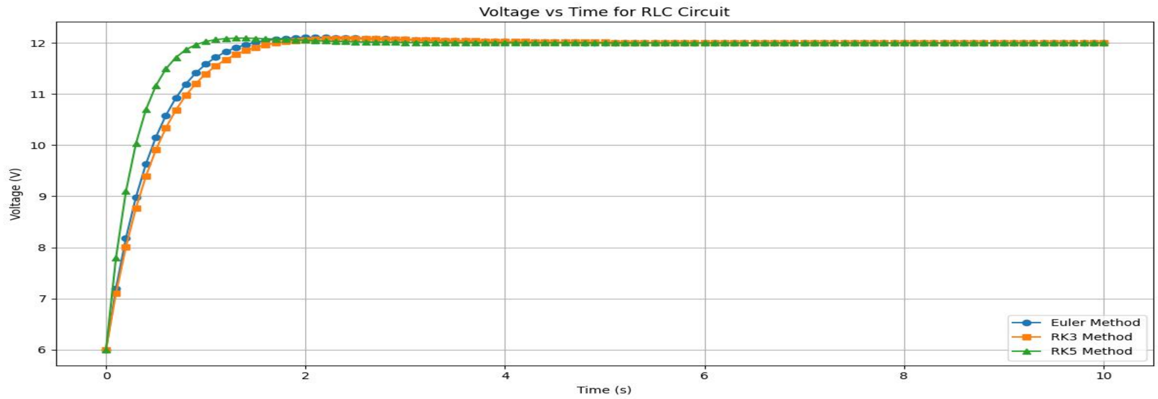

Fig. 2: It shows the resistance effect for critically damped (R=$\sqrt{0.5}$, C=0.5, L=1)

Table 3: Comparison results of the three numerical methods (Exact, Euler, RK3 and RK5) critically damped R=$\sqrt{0.5}$, C=0.5, L=1 for the Voltage

Time (s)	Voltage (Exact)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)
0.0	6.00	6.00	6.00	6.00
0.1	7.12	7.20	7.16	7.12
0.2	8.22	8.09	8.28	8.22
0.3	9.23	8.69	9.35	9.23
0.4	10.20	9.02	10.27	10.20
0.5	11.05	9.19	11.06	11.05
0.6	11.67	9.22	11.71	11.67
0.7	12.17	9.13	12.23	12.17
0.8	12.53	8.94	12.62	12.53
0.9	12.77	8.68	12.88	12.77
1.0	13.00	8.36	13.03	13.00

Table 4: Comparison results of the three numerical methods (Euler, RK3 and RK5) critically damped R=$\sqrt{0.5}$, C=0.5, L=1 for the Current

Time (s)	Current (Exact)	Current (Euler)	Current (RK3)	Current (RK5)
0.0	12.00	12.00	12.00	12.00
0.1	11.07	10.71	11.05	11.07
0.2	9.23	9.52	9.24	9.23
0.3	7.72	8.31	7.52	7.72
0.4	6.33	7.13	6.07	6.33
0.5	4.97	5.97	4.78	4.97
0.6	3.92	4.83	3.80	3.92
0.7	3.04	3.69	2.91	3.04
0.8	2.31	2.54	2.19	2.31
0.9	1.70	1.37	1.62	1.70
1.0	0.14	0.18	0.13	0.14

These findings show that the RK5 approach is the most accurate, especially when considering a greater number of iterations, compared to the Euler, RK3, and RK5 methods. The RK5 approach usually yields an approximate result.

Table 5: Comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=0.1, C=0.5, L=1 for the Voltage

Time (s)	Voltage (Exact)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)
0.0	6.00	6.00	6.00	6.00
0.1	7.122	7.20	7.12	7.122
0.2	8.244	-0.96	8.24	8.244
0.3	9.356	-4.56	9.23	9.356
0.4	10.457	-9.08	10.22	10.457
0.5	11.549	-14.61	11.17	11.549
0.6	12.631	-21.24	12.08	12.631
0.7	13.704	-28.97	12.96	13.704
0.8	14.767	-37.89	13.82	14.767
0.9	15.821	-48.05	14.66	15.821
1.0	16.865	-59.50	15.48	16.865

Table 6: Comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=0.1, C=0.5, L=1 for the Current

Time (s)	Current (Exact)	Current (Euler)	Current (RK3)	Current (RK5)
0.0	12.00	12.00	12.00	12.00
0.1	10.285	-10.80	10.25	10.285
0.2	9.056	-35.04	8.88	9.056
0.3	8.00	-45.20	7.71	8.00
0.4	7.12	-55.32	6.77	7.12
0.5	6.42	-65.41	6.01	6.42
0.6	5.86	-75.49	5.37	5.86
0.7	5.42	-85.55	4.83	5.42
0.8	5.08	-95.59	4.36	5.08
0.9	4.82	-105.63	3.96	4.82
1.0	4.64	-115.65	3.62	4.64

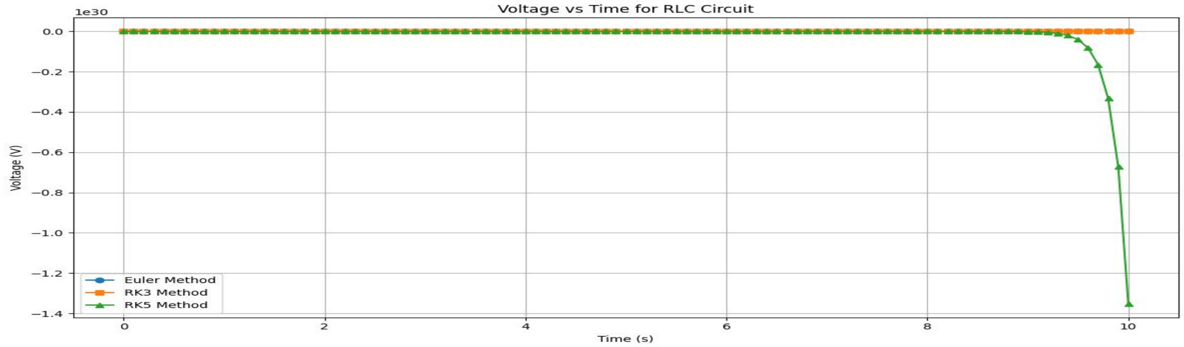

Fig. 3: the resistance effect for under damped (R=0.1, C=0.5, L=1)

Table 7: Comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=1, C=0.1, L=1 for the Voltage

Time (s)	Voltage (Exact)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)
0.0	6.00	6.00	6.00	6.00
0.1	7.15	7.20	7.18	7.15
0.2	8.21	8.10	8.28	8.21
0.3	9.18	8.79	9.30	9.18
0.4	10.08	9.31	10.21	10.08
0.5	10.90	9.70	11.04	10.90
0.6	11.64	9.99	11.77	11.64
0.7	12.30	10.19	12.39	12.30
0.8	12.89	10.34	12.91	12.89
0.9	13.41	10.44	13.34	13.41
1.0	13.86	10.51	13.69	13.86

Table 8: The comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=1, C=0.1, L=1 for the Current

Time (s)	Current (Exact)	Current (Euler)	Current (RK3)	Current (RK5)
0.0	12.00	12.00	12.00	12.00
0.1	9.08	9.00	9.10	9.08
0.2	6.86	6.90	6.82	6.86
0.3	5.21	5.21	5.03	5.21
0.4	4.02	4.04	3.72	4.02
0.5	2.94	2.93	2.73	2.94
0.6	1.98	1.89	1.99	1.98
0.7	1.38	0.91	1.44	1.38
0.8	0.91	0.00	1.02	0.91
0.9	0.57	-0.83	0.69	0.57
1.0	0.32	-1.60	0.42	0.32

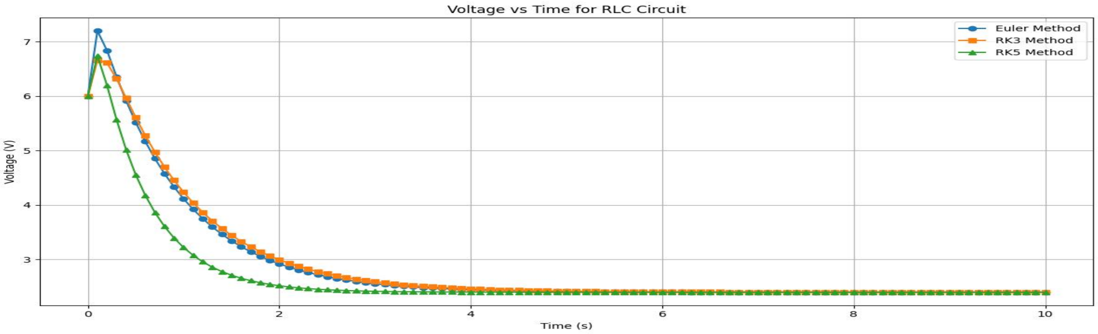

Fig: 4 It shows the capacitance effect for under damped (R=1, C=0.1, L=1)

The Euler Method yields a crude estimate that differs noticeably from the precise result.

The RK3 method has more intermediate steps and is more accurate than the Euler method.

Given its intricacy, the RK5 Method typically yields findings that are highly accurate and near to the precise solution.

Table 9: comparison results of the three numerical methods (Euler, RK3 and RK5) over damped R=1, C=0.25, L=1 for the Voltage

Time (s)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)	Voltage (Exact)
0.0	6.00	6.00	6.00	6.00
0.1	7.20	7.18	7.19	7.18
0.2	8.16	8.28	8.30	8.29
0.3	8.93	9.30	9.33	9.31
0.4	9.54	10.21	10.26	10.24
0.5	10.03	11.04	11.09	11.07
0.6	10.42	11.77	11.84	11.82
0.7	10.74	12.39	12.49	12.47
0.8	10.99	12.91	13.05	13.03
0.9	11.20	13.34	13.52	13.50
1.0	11.37	13.69	13.91	13.89

Table 10: comparison results of the three numerical methods (Euler, RK3 and RK5) over damped R=1, C=0.25, L=1 for the Current

Time (s)	Current (Euler)	Current (RK3)	Current (RK5)	Current (Exact)
0.0	12.00	12.00	12.00	12.00
0.1	9.60	9.70	9.72	9.72
0.2	7.68	7.71	7.73	7.73
0.3	6.14	6.08	6.11	6.11
0.4	4.91	4.75	4.76	4.76
0.5	3.93	3.67	3.68	3.68
0.6	3.14	2.79	2.81	2.81
0.7	2.50	2.07	2.10	2.10
0.8	1.96	1.50	1.52	1.52
0.9	1.50	1.05	1.08	1.08
1.0	1.11	0.70	0.73	0.73

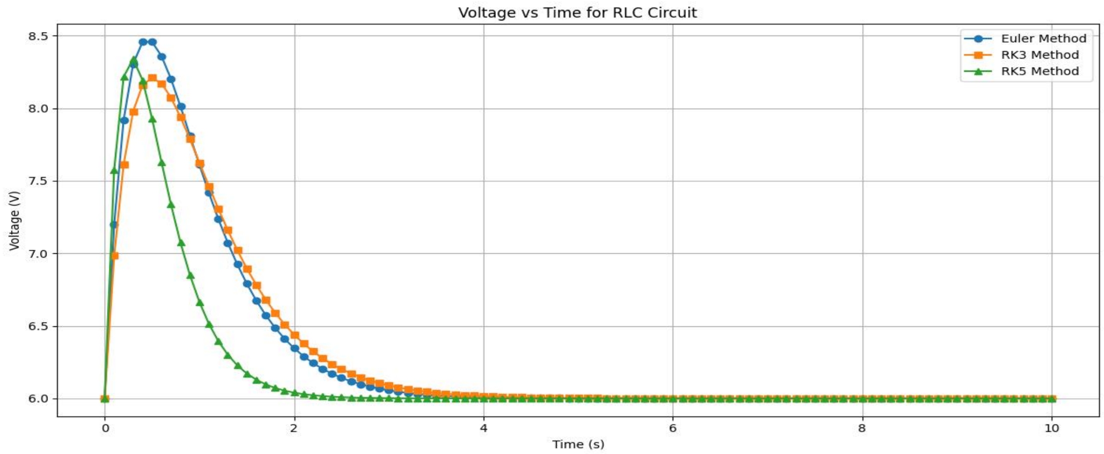

Fig. 5: The capacitance effect for over damped (R=1, C=0.25, L=1)

The Euler Method yields a crude estimate that differs noticeably from the precise result.
The RK3 method has more intermediate steps and is more accurate than the Euler method.
Given its intricacy, the RK5 Method typically yields findings that are highly accurate and near to the precise solution.

Table 11: Comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=1, C=0.25, L=0.5 for the Voltage

Time (s)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)	Voltage (Exact)
0.0	6.00	6.00	6.00	6.00
0.1	7.20	7.16	7.18	7.18
0.2	8.12	8.24	8.29	8.27
0.3	8.81	9.22	9.31	9.30
0.4	9.32	10.09	10.23	10.22
0.5	9.67	10.86	11.07	11.06
0.6	9.91	11.54	11.83	11.82
0.7	10.06	12.12	12.51	12.50
0.8	10.14	12.62	13.11	13.10
0.9	10.16	13.03	13.63	13.62
1.0	10.15	13.37	14.08	14.06

Table 12: Comparison results of the three numerical methods (Euler, RK3 and RK5) under damped R=1, C=0.25, L=0.5 for the Current

Time (s)	Current (Euler)	Current (RK3)	Current (RK5)	Current (Exact)
0.0	12.00	12.00	12.00	12.00
0.1	9.20	9.26	9.30	9.30
0.2	6.88	7.10	7.13	7.13
0.3	4.94	5.34	5.38	5.38
0.4	3.33	3.91	3.95	3.95
0.5	1.97	2.73	2.78	2.78
0.6	0.85	1.76	1.81	1.81
0.7	-0.13	0.96	1.01	1.01
0.8	-0.98	0.31	0.35	0.35
0.9	-1.70	-0.23	-0.18	-0.18
1.0	-2.30	-0.69	-0.63	-0.63

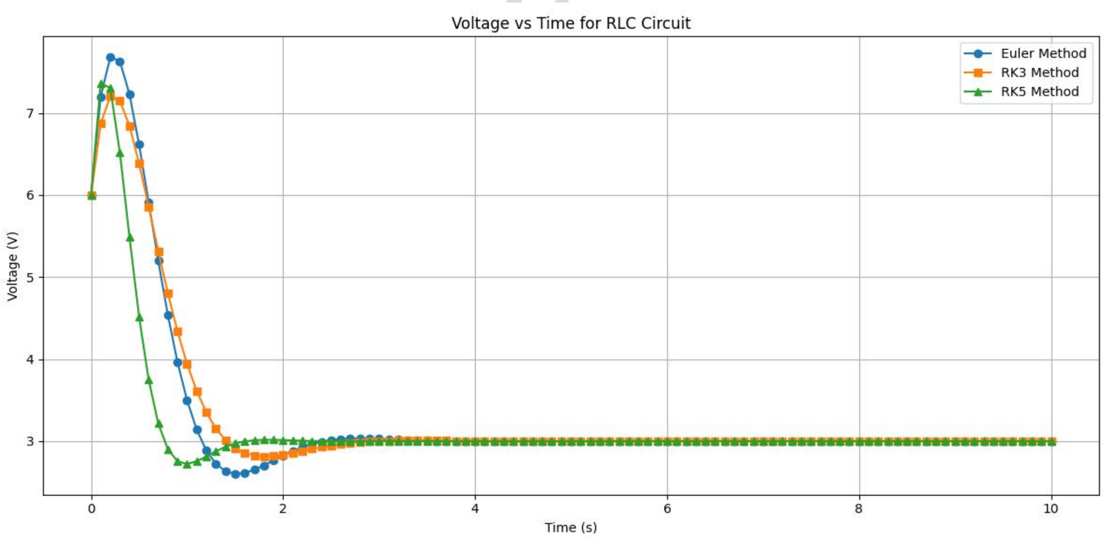

Fig.6: It shows the inductance effect for under damped (R=1, C=0.25, L=0.5)

The Euler Method yields a crude estimate that differs noticeably from the precise result.
The RK3 method has more intermediate steps and is more accurate than the Euler method.
Given its intricacy, the RK5 Method typically yields findings that are highly accurate and near to the precise solution.

Table 13: shows the comparison results of the three numerical methods (Exact, Euler, RK3 and RK5) critically damped R=1, C=0.25, L=0.5 for the Voltage

Time (s)	Voltage (Euler)	Voltage (RK3)	Voltage (RK5)	Voltage (Exact)
0.0	6.00	6.00	6.00	6.00
0.1	7.20	7.24	7.28	7.28
0.2	8.16	8.32	8.37	8.35
0.3	8.89	9.25	9.28	9.26
0.4	9.43	10.04	10.06	10.04
0.5	9.80	10.71	10.72	10.72
0.6	10.03	11.28	11.29	11.29
0.7	10.15	11.76	11.76	11.76
0.8	10.20	12.15	12.14	12.14
0.9	10.18	12.46	12.45	12.45
1.0	10.12	12.70	12.69	12.69

Table 14: shows the comparison results of the three numerical methods (Exact Euler, RK3 and RK5) critically damped R=1, C=0.25, L=0.5 for the Current

Time (s)	Current (Euler)	Current (RK3)	Current (RK5)	Current (Exact)
0.0	12.00	12.00	12.00	12.00
0.1	9.60	9.68	9.72	9.72
0.2	7.36	7.60	7.65	7.65
0.3	5.34	5.77	5.82	5.82
0.4	3.50	4.15	4.20	4.20
0.5	1.83	2.72	2.77	2.77
0.6	0.31	1.44	1.49	1.49
0.7	-1.07	0.29	0.34	0.34
0.8	-2.31	-0.72	-0.67	-0.67
0.9	-3.42	-1.56	-1.51	-1.51
1.0	-4.39	-2.27	-2.22	-2.22

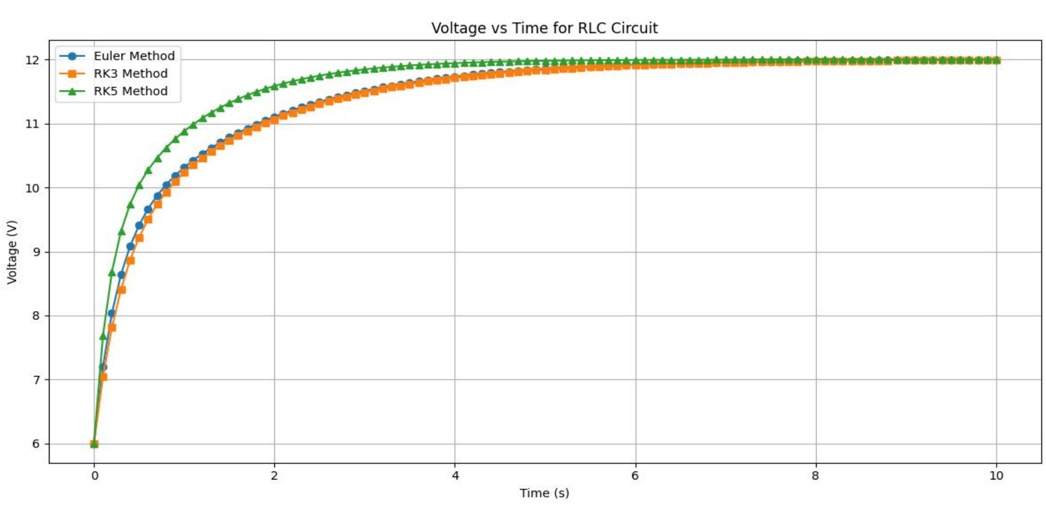

Fig: 7 It shows the inductance effect for critically damped (R=1, C=0.25, L=2)

The Euler method yields a crude estimate that differs noticeably from the precise result.

The RK3 method has more intermediate steps and is more accurate than the Euler method.

Given its intricacy, the RK5 method typically yields findings that are highly accurate and near to the precise solution.

Euler results in the lowest accuracy while RK5 provides the highest and RK3 method is intermediate. Since, Euler method is first order, it can easily diverge or give inaccurate findings. In contrast, RK5 and RK3 method has better stability log (time) over a longer period computation.

Although, Euler method is the simplest and fastest to calculate, it is not accurate. RK3 is an intermediate between RK5 and Euler in terms of fidelity to diesel operation. The most accurate of all, but also most time-consuming method is RK5.

4. Discussion and Analysis

This section will concentrate on the comparison of three numerical methods: Euler, RK3, and RK5, with their application in solving the differential equations describing a damped parallel RLC circuit. The general effect of resistance R, inductance L, and capacitance C variations on the circuit behavior will be studied to thoroughly understand the system dynamics.

Euler Method

The Euler Method, easy to compute and implement, produces the largest accumulation of error as the number of iterations increases. First-order solution methods tend to have a large difference between the produced solution and the one that would have been found if a continuous or very finely discretized model had been used, especially in systems that exhibit high-frequency oscillations and rapid transients. This technique is proposed for rough initial analysis or in cases where ease of calculation is more desirable than accuracy.

RK3 Method

The RK3 Method yields a good compromise between computational performance and accuracy. It is inherently third-order, thus producing results very close to the true solution. RK3 has very little deviation from RK5, but RK5 order is much more computationally efficient; thus, it is useful whenever moderately accurate solutions are required with low computational cost.

RK5 Method

Across all performance metrics, the RK5 Method exhibited the best results. Its fifth-order nature guarantees minimal error accumulation over long periods. This method is particularly effective in capturing the transient response, i.e., initial oscillations followed by steady-state response, thus making it well suited for high-precision applications or when system precision is imperative.

The $V(t)$ vs. time and $I_L(t)$ vs. time plots represent the transients and steady states:

Voltage Plot: The RK5 method properly tracks the expected behavior by capturing the transient's oscillatory effects with a good damping effect. In contrast, the Euler method deviates significantly, while RK3 achieves mediocrity.

Inductor Current Plot: Similar trends are observed, with the RK5 method most accurately depicting current behavior, followed by RK3 and Euler.

Effects of Varied R, L and C

Increased R: This would mean strong damping of the oscillations because resistance now becomes much stronger. In this case, when the steady state was reached, there were lower peak voltages and currents. This is usually good for stability, but it means that the circuit would not sustain oscillations for a long time.

Decreased R: This decreases the damping, allowing longer amplitude oscillations. Of course, this is good for energy storage but may result in an instability or an inefficient energy dissipation in a practical case.

Increased L: Increase in inductance will mean slowness in the circuit's response because, at this stage, the inductor is more effective in opposing the changes in current. This leads to lower oscillation frequencies and thereby smooth transitions. On the downside, it increases the time taken for the steady-state conditions to be reached.

Decreased L: Reduced inductance causes faster responses with higher frequencies of oscillation and sharper transients. This could be appropriate for high-speed applications but would become problematic in systems that require controlled responses.

Increased C: The increase in capacitance leads to storing more energy and having lower frequencies of oscillation: thus, longer transients. This enhances the ability of the system for energy storage but reduces its responsiveness to changes.

Decreased C: Decrease in capacitance leads to higher frequencies of oscillation and shorter response times. This increases the responsiveness of the circuit but diminishes its effectiveness in smoothing out voltage variations and may lead to instability.

Conclusion and Future Work

Though simpler and easier to use, the Euler approach is less accurate than the RK3 approaches. Because of its higher-order accuracy, the RK3 technique yields results that are quite similar to the real answer. The combined graph of the methods illustrates the variations in accuracy.

An easy way to approximate the behavior of a parallel RLC circuit is to use the Euler method. Iteratively updating the voltage and current numbers while discretizing time allows us to see the circuit's dynamic response. The findings show that the voltage should decrease exponentially and that the inductor's current should rise accordingly. An accurate method for approximating the behavior of a parallel RLC circuit is the RK3 method. Iteratively updating the voltage and current numbers while discretizing time allows us to see the circuit's dynamic response. The outcomes show the anticipated exponential.

For solving the parallel RLC circuit equation, the RK5 method is preferred for high accuracy, especially for critical applications. The RK3 method provides a good balance of accuracy and computational efficiency. The Euler method, while easy to implement, may not be suitable for precise calculations over longer periods. Here's an expanded version of the text into a full paragraph:

The third objective of this research aims to provide deeper insights into the research question by evaluating the accuracy, stability, and computational effort required by each of the three models. Through detailed numerical analysis, this objective assesses how effectively each model performs in terms of precision and robustness under various conditions. By comparing these metrics, the research seeks to highlight the strengths and limitations of each model, offering a clearer understanding of their applicability and efficiency in different contexts. This analysis not only demonstrates the comparative accuracy of each model but also sheds light on the computational demands, helping to determine the most suitable model based on specific research needs.

The Euler Method provides a basic estimate that significantly deviates from the exact solution, offering a low level of accuracy due to its simplicity and single-step approach. The RK3 method, which involves more intermediate steps, achieves greater accuracy than the Euler Method and serves as a middle-ground approach between simplicity and precision. On the other hand, the RK5 method, with its increased complexity and additional steps, delivers highly precise results that closely approximate the exact solution.

5.2 Future Work

Building on this study, there are a number of directions that future research may go:

Methods for adaptive step size: To maximize precision and computing effectiveness, put adaptive step size techniques into practice and compare them. These techniques use error estimations to dynamically modify the step size. Higher-level techniques: To further increase accuracy, look into implementing multi-step methods (such Adams-Bashforth, Adams-Moulton) or even higher-order Runge-Kutta methods. Analysis of Stability: To comprehend the behavior of the numerical algorithms under various parameters and conditions, perform a thorough stability study. Applications in Real Time: Examine how these numerical techniques can be used in real-time systems where accuracy and computing efficiency are crucial, including control systems.

Examine how these numerical approaches can be made more computationally efficient to handle large-scale problems by exploring the usage of parallel computing techniques.

Analysis and correction of errors: create and implement error correction strategies that enhance the performance of lower-order approaches, such as the Euler method, and increase their ability to compete with higher-order techniques. Hybrid approaches: Investigate hybrid approaches that maximize accuracy and efficiency by combining the best features of various numerical techniques.

Through the pursuit of these forthcoming avenues, scholars might persistently augment the efficacy and relevance of numerical techniques in resolving intricate differential equations throughout diverse domains. Researchers may extend this concept to fractional integral equations [34].

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