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ON RELATION OF ALTERNATIVE LA-SEMIGROUP

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Abstract
In this paper, we establish the relation of alternative LA-semigroup with existence subclasses of LA-semigroup. We prove that under what condition a left alternative LA-semigroup becomes right alternative, flexible, left, right and middle nuclear square LA-semigroups. By the use of Modern mathematical technique GAP and Mace4 we construct a non-associative example that left alternative is neither contain in right alternative, flexible, left, right and middle nuclear square LA-semigroups. Although left(right) alternative LA-semigroup is non-associative, but we prove that the combination of T^1 and T^4- LA-semigroups it becomes associative. We also find the relation of right alternative, Jordan, stein, LA,LC, RC, middle and right nuclear square LA-semigroups.*

INTRODUCTION

In[1] a non-associative structure introduced by Kazim and Naseeruddin called **LA-Semigroup** satisfying the identity $(ij)k = (kj)i$ known **left invertive law**. In the literature various researcher introduced the same structure with different name like, left almost semigroup (LA-semigroup) [1, 2, 3, 4, 5], Abel-Grassmann's groupoid (AG-groupoid) [6, 7, 8, 9], right modular groupoid [10] and left invertive groupoid [11]. In this dissertation, the term LA-semigroup will be used for this structure throughout and will be represented by L. A groupoid satisfying the identity $a(bc) = c(ba)$ known right invertive law is called a right AG-groupoid (or RA-semigroup). L always satisfies the *medial law*:

$$(ab)(cd) \quad = \quad (ac)(bd)\ \forall a,b,c,d \in L. \qquad (2.1)$$

An element $e \in L$ is called its *left identity* if $ea = a$, $\forall a \in L$. It is proved in [3] that left identity if exists in L then it is unique. An L with left identity e always satisfies the *paramedial law*: $ab \cdot cd = db \cdot ca, \forall a ,b, c, d \in L$ [10]. An L whose all elements are idempotent is called *LA-band* [12]. An L which satisfy $(aa)a = a(aa) = a$, $\forall a \in L$, is called *LA-3-band* [13]. An L which satisfies the two equivalent identities $(ab)c = b(ac)$ and $(ab)c = b(ca)$ $\forall a, b, c \in L$, is called *LA*groupoid* [14]. An L which satisfies $a(bc) = b(ac)$, $\forall a, b, c \in L$, is called *LA**groupoid* [15]. An LA-semigroup with the left identity is called *LA-monoid*.

Every LA-monoid is *LA***-groupoid . An element a of L is called *left cancellative*: if $ax = ay \Rightarrow x = y\ \forall x, y \in L$. Similarly an element a of L is called *right cancellative*: if $xa = ya \Rightarrow x = y\ \forall x, y \in L$ and a is called *cancellative*: if it is both left and

right cancellative. L is called *left cancellative (resp. right cancellative, cancellative)***:** if every element of L is left cancellative (resp. right cancellative, cancellative)[16]. In [17] Q. Mushtaq and S. M. Yousaf defined *locally associative LA-Semigroup* and proved that in locally associative LA-semigroup L the exponent laws hold.
In 2006 a new era of LA-semigroup initiated when Q. Mushtaq and M. Khan [18] introduced ideals in LA-semigroup and especially when fuzzification of LA-semigroups was done in [19]. This really attracted various researchers to LA-group whose effect we can see in 2011 and 2012 that numerous papers were published and some of them even came online. Ideals in particular subclasses such as LA-band and LA*-semigroup were discussed in [20, 21], and decomposition of a locally associative LA**-semigroup in [22]. Later on in 2012, LA-semigroup were enumerated up to order 6 [23] and classification of LA-semigroup was done[24, 25]. Muhammad Rashad in his Ph.D. thesis introduced new subclasses of LA-groupoid[26]. He also enumerate them up to order 6. The idea of LA-semigroup is extended to the class of Modulo LA-semigroup, matrix modulo-LA-semigroup and polynomial modulo LA-semigroup in [27, 28, 29]. The properties of T^1, T^2 and T^4-LA-Semigroups are investigated in[30]. Relation of right alternative and nuclear square Ag-groupoid are find in [31]. The properties new subclasses of Bi-Commutative, Stein, Right abelian distributive, Unar and Left Transitive LA-Semigroups are discussed in [32, 33, 34, 35, 36, 37].

Basic Definitions and Notions

The following definition are frequently use in this article
(def-1) A groupoid L is called LA-semigroup, if for $a, b, c \in L$, we have
$$ab \cdot c = cb \cdot a \quad (3.1)$$
(def-2) An LA-semigroup L is called left alternative LA-semigroup, if for $a, b \in L$, we have
$$aa \cdot b = a \cdot ab \quad (3.2)$$
(def-3) An LA-semigroup L is called T^1-LA-semigroup, if for $a, b, c, d \in L$, we have
$$ab = cd \Rightarrow ac = bd \quad (3.3)$$
(def-4) An LA-semigroup L is called forward T^4-LA-semigroup denoted by T_f^4 if
$$ab = cd \Rightarrow ad = cb \quad \forall a, b, c, d \in L, \quad (3.4)$$
(def-5) LA-semigroup L is called backward T^4-LA-semigroup, denoted by T_b^4, if
$$ab = cd \Rightarrow da = bc \quad \forall a, b, c, d \in L. \quad (3.5)$$
(def-6) An LA-semigroup L is called T^4-LA-semigroup if it is both forward and backward T^4-LA-semigroup.
(def-7) An LA-semigroup L is called right alternative LA-semigroup, if for $a, b \in L$, we have
$$b \cdot aa = ba \cdot a \quad (3.6)$$
(def-8) An LA-semigroup L is called an LA-Band, if for $a \in L$, we have
$$a \cdot a = a \quad (3.7)$$
(def-9) An LA-semigroup L is called left nuclear square LA-semigroup, if for $a,b,c \in L$, we have
$$a^2 \cdot bc = a^2b \cdot c \quad (3.8)$$
(def-10) An LA-semigroup L is called right nuclear square LA-Semigroup, if for $a,b,c \in L$, we have
$$a \cdot bc^2 = ab \cdot c^2 \quad (3.9)$$
(def-11) An LA-semigroup L is called middle nuclear square LA-semigroup, if for $a,b,c \in L$, we have
$$a \cdot b^2c = ab^2 \cdot c \quad (3.10)$$
(def-12) An LA-semigroup L is called Jordan LA-Semigroup, if for $a,b,c \in L$, we have
$$a \cdot b^2c = b^2 \cdot ac \quad (3.11)$$
(def-13) An LA-semigroup L is called paramedial LA-Semigroup, if we have
$$ab \cdot cd = db \cdot ca \quad \forall a,b,c,d \in L, \quad (3.12)$$
(def-14) An LA-semigroup L is called flexible LA-Semigroup, if for $a,b \in L$, we have
$$ab \cdot a = a \cdot ba \quad (3.13)$$

(def-15) An LA-semigroup L is called stein LA-Semigroup, if for $a,b,c \in L$, we have

$$a \cdot bc = bc \cdot a \quad (3.14)$$

(def-16) An LA-semigroup L is called LC-LA-Semigroup, if for $a,b,c \in L$, we have

$$ab \cdot c = ba \cdot c \quad (3.15)$$

(def-17) An LA-semigroup L is called RC-LA-Semigroup, if for $a,b,c \in L$, we have

$$a \cdot bc = a \cdot cb \quad (3.16)$$

(def-18) An element $e \in L$ is called left identity, if for all $a \in L$, we have

$$e \cdot a = a \quad (3.17)$$

(def-19) An LA-semigroup L is called RLA-Semigroup or self-dual AG-groupoid, if for $a,b,c \in L$, we have

$$a \cdot bc = c \cdot ba \quad (3.18)$$

(def-20) An LA-semigroup L is called locally associative LA-Semigroup, if for $a \in L$, we have

$$a \cdot aa = aa \cdot a \quad (3.19)$$

(def-21) An LA-semigroup L is called LA-monoid, if L contain left identity

$$e \cdot a = a \quad \forall\, a \in G \quad (3.20)$$

(def-22) An LA-semigroup L is called LA^*-semigroup, if $\forall a,b,c \in L$,

$$ab \cdot c = b \cdot ac \quad (3.21)$$

Relation of left(right) alternative LA-semigroup and locally associative LA-semigroup

In this section we prove the relation of left(right) alternative LA-semigroup with locally associative LA-semigroup.

Lemma 1. *Every left(right) alternative LA-Semigroup is locally associative.*

Proof. Let A be left(right) alternative LA-Semigroup. The for any $a \in$ A, using (3.2), (3.6)

$$aa \cdot a \overset{(3.2)}{=} a \cdot aa \quad \overset{(3.6)}{(a \cdot aa = aa \cdot a)}$$

Hence, A is locally associative LA-Semigroup.

Relation of left alternative LA-semigroup and LA*−semigroup

Now here we prove that every LA*−semigroup is left alternative LA-semigroup, but not right alternative LA-semigroup.

Lemma 2. *Every LA*-semigroup is left alternative LA-semigroup.*

Proof. Let A be an LA*-semigroup. Then for any $a,b \in$ A. Then we have by (3.21)

$$aa \cdot b = a \cdot ab$$

Hence, A is left transitive LA-semigroup.

Now the following Cayley's table is LA*-semigroup, but not right alternative LA-semigroup.

*	1	2	3	4	5	6
1	3	3	3	3	6	3
2	4	5	3	3	3	3
3	3	3	3	3	3	3
4	3	6	3	3	3	3
5	3	3	3	3	3	3
6	3	6	3	3	3	3

Because

$1 * (2 * 2) \neq (1 * 2) * 2 \Rightarrow 1 * 5 \neq 3 * 2 \Rightarrow 6 \neq 3$

Relation of Left Alternative LA-Semigroup satisfying the property of self-dual LA-Semigroup with other subclasses of LA-Semigroup

In this section we show that a left alternative LA-Semigroup L satisfying the property of self-dual LA-Semigroup. Then L is
Right alternative LA-Semigroup.
Left nuclear square LA-Semigroup.
Right nuclear square LA-Semigroup.
Middle nuclear square LA-Semigroup.
Before proving the result, first we construct an example in the following Cayley's table that neither left alternative LA-Semigroup nor self-dual LA-Semigroup is right alternative, left, right and middle nuclear squares LA-Semigroups.
Example 1. *(a) Cayley's table(i) is left alternative LA-Semigroup, which is not left nuclear square LA-Semigroup.*

Cayley's table(ii) is self-dual LA-Semigroup, which is non of the following (i) Right Alternative LA-Semigroup.
Right Nuclear Square LA-Semigroup.
Middle Nuclear Square LA-Semigroup.
Left nuclear square LA-Semigroup.
left Alternative LA-Semigroup.

Cayley's table(iii) is Left alternative LA-Semigroup, which is none of the following
Right Alternative LA-Semigroup.
Right Nuclear Square LA-Semigroup.
Middle Nuclear Square LA-Semigroup.
Self-dual LA-Semigroup

*	1	2	3	4	5	6	7
1	3	4	1	7	7	7	7
2	5	4	6	7	7	7	7
3	1	7	2	7	7	7	7
4	7	7	7	7	7	7	7
5	7	4	4	7	7	7	7
6	4	4	7	7	7	7	7
7	7	7	7	7	7	7	7

Table(*i*)

*	1	2	3	4
1	2	3	1	4
2	4	1	3	2
3	3	2	4	1
4	1	4	2	3

Table(*ii*)

*	1	2	3	4
1	3	3	4	4
2	4	2	4	4
3	4	4	4	4
4	4	4	4	4

Table (iii)

Now we prove the result

Theorem 1. *Let L be a left alternative LA-Semigroup satisfying the property of self-dual LA-Semigroup. Then L is*
Right alternative LA-Semigroup.
Left nuclear square LA-Semigroup.
Right nuclear square LA-Semigroup.
Middle nuclear square LA-Semigroup.

Proof. Let L be a left alternative LA-Semigroup having self-dual LA-Semigroup property. Then ∀, $a,b,c \in$ L, and by identities, (3.1), (3.2), and (3.18) we have

$$a \cdot bb = b \cdot ba = bb \cdot a = ab \cdot b$$

Hence, L is right alternative LA-Semigroup.

Then by identities, (2.1), (3.1), (3.2), and (3.18) we have

$$a^2 \cdot bc = a(a \cdot bc) = a(c \cdot ba) = ba \cdot ca = bc \cdot aa = (aa \cdot c)b$$
$$= (a \cdot ac)b = (b \cdot ac)a = (c \cdot ab)a = (a \cdot ab)c = (aa \cdot b)c$$

thus, $a^2 \cdot bc = a^2b \cdot c$. So, L is left nuclear square LA-Semigroup.

Again, by the use of defined identities, (3.1), (3.2), and (3.18) we have

$$a \cdot bc^2 = cc \cdot ba = c(c \cdot ba) = ba \cdot cc = (cc \cdot a)b = (c \cdot ca)b$$
$$(b. ca)c = (a \cdot cb)c = (c \cdot cb)a = (cc \cdot b)a = ab \cdot c^2$$

thus, $a \cdot bc^2 = ab \cdot c^2$.

Therefore, L is right nuclear square LA-semigroup.

Similarly, by identities, (2.1), (3.1), (3.2), and (3.18)

$$a \cdot b^2c = a(b \cdot bc) = a(c \cdot bb) = bb \cdot ca = b(b \cdot ca) = b(a \cdot cb)$$
$$=cb \cdot ab = ca \cdot bb = (bb \cdot a)c = (b \cdot ba)c = (a \cdot bb)c = ab^2 \cdot c$$

thus, $a \cdot b^2c = ab^2 \cdot c$

Hence, L is middle nuclear square LA-semigroup.

Relation of Left Alternative LA-semigroup with flexible LA-semigroup

In this section we find that under what condition the left alternative LA-semigroup becomes flexible LA-semigroup. We construct an example in which we investigate that neither every left alternative nor every LA-Monoid is flexible, also neither flexible nor LA-Monoid is left alternative LA-semigroup

Example 2. *(a) Cayley's table(i) is left Alternative LA-semigroup, which is neither LA-Monoid nor flexible LA-semigroup. Cayley's table(ii) is flexible LA-semigroup, which is neither LA-monoid nor left alternative LA-semigroup.*

Cayley's table(iii) is LA-monoid, which is neither flexible LA-semigroup nor left alternative LA-semigroup.

*	1	2	3	4
1	3	3	3	2
2	4	3	3	3
3	3	3	3	3
4	3	1	3	3

Table(i)

*	1	2	3
1	3	2	3
2	3	3	3
3	3	3	3

Table(ii)

*	1	2	3
1	1	2	3
2	3	1	2
3	2	3	1

Table (iii)

Now in the following theorem we show that every left alternative LA-monoid is flexible LA-semigroup.

Theorem 2. *Let* L *be a left alternative LA-semigroup with left identity. Then* L *is flexible LA-semigroup.*

Proof. Let L be a left alternative LA-semigroup with left identity, the by use of identities (2.1), (3.1) and (3.2) we have

$$ab \cdot a = (e(ab)) \cdot a = (a \cdot ab)e = (aa \cdot b)e = eb \cdot aa = ea \cdot ba = a \cdot ba$$

Hence, L is flexible LA-semigroup.

Relation of Left alternative T^1-LA-semigroup with Flexible LA-semigroup

In this section we show that a left alternative LA-semigroup satisfying T^1-LAsemigroup is flexible LA-semigroup. Before proving this result, first we construct an example which show that neither of left alternative, T^1 and flexible LA-semigroups are containing in each other.

Example 3. *(a) Cayley's table(i) is left Alternative LA-semigroup, which is*

(i) neither T^1-LA-semigroup because

$1 * 2 = 4 * 3 \not\Rightarrow 1 * 4 = 2 * 3 \Rightarrow 2 \neq 3$

(ii) nor flexible LA-semigroup. because

$(2 * 4) * 2 \neq 2 * (4 * 2) \Rightarrow 3 * 2 \neq 2 * 1 \Rightarrow 3 \neq 4$

Cayley's table(ii) T^1-LA-semigroup, which is

(i) neither flexible LA-semigroup, because
(1 * 2) * 1 ≠ 1 * (2 * 1) ⇒ 3 * 1 ≠ 1 * 1 ⇒ 3 ≠ 2
(ii) nor left alternative LA-semigroup, because
(1 * 1) * 3≠ 1 * (1 * 3) ⇒ 2 * 3 ≠ 1 * 1 ⇒ 3 ≠2
Cayley's table(iii) is flexible LA-semigroup, which is
(i) neither T^1*-LA-semigroup because*
1* 3 = 2* 3≠> 1 * 2 = 3 * 3 ⇒ 2 ≠ 3
(ii) nor left alternative AG-Groupoids. Because
(1*1) * 2 ≠ 1 * (1 * 2) ⇒ 3 * 2 ≠ 1 * 2 ⇒ 3 ≠2

*	1	2	3	4
1	3	3	3	2
2	4	3	3	3
3	3	3	3	3
4	3	1	3	3

Table (i)

*	1	2	3
1	2	3	1
2	1	2	3
3	3	1	2

Table (ii)

?	1	2	3
1	3	2	3
2	3	3	3
3	3	3	3

Table (iii)

Now we prove the result that every left alternative LA-semigroup satisfying the property of T^1-LA-semigroup is flexible LA-semigroup.

Theorem 3. *Every left alternative* T^1*-LA-semigroup is flexible LA-semigroup.*

Proof. Let L be a left alternative T^1-LA-semigroup. Then by use of identities (3.1), (3.2) and (3.3) we have

$$a \cdot b = ba \cdot a \Rightarrow a \cdot ab = ba \cdot a$$
$$\Rightarrow a \cdot ba = ab \cdot a$$

Hence, L is flexible LA-semigroup.

Relation of Left Alternative LA-semigroup and Semigroup

In this section we proved that left alternative LA-semigroup L becomes semigroup if it satisfies any one of the property that
(a) T^1- LA-semigroup. (b) T^4- LA-semigroup.
But before we construct an example of non-associative and non-commutative left alternative, T^1 and T^4-LA-semigroups.

Example 4. *The non-associative and non-commutative left alternative LA-semigroup is given in Table (i).*
T^1- LA-semigroup is given in Table (ii).
T^4- LA-semigroup is given in Table (iii).

?	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	1	1	4

Table(i)

*	1	2	3
1	1	1	1
2	1	1	3
3	1	2	1

Table(ii)

*	1	2	3	4
1	1	2	3	4
2	2	1	4	3
3	4	3	2	1
4	3	4	1	2

(Table (iii))

Corollary 1. [30] *Every T^4-LA-semigroup is paramedial LA-Semigroup.*
Now we have the following result.

Theorem 4. *Let* L *be a left alternative LA-semigroup. Then it becomes semigroup if any one of the following holds*
(i) L *is T^1- LA-semigroup.* *(ii)* L *is T^4- LA-semigroup.*

Proof. Let L be a left alternative LA-semigroup. Then for $\forall, a, b, c \in$ L and by use of definitions (2.1), (3.1). (3.2) and (3.3), we have

$$ab \cdot c = cb \cdot a \quad \Rightarrow \quad ab \cdot cb = ca$$
$$ac \cdot bb = ca \quad \Rightarrow \quad ac \cdot c = bb \cdot a$$
$$cc \cdot a = ab \cdot b \quad \Rightarrow \quad c \cdot ca = ab \cdot b,$$

thus, $c(ab) = (ca)b$.

Hence, L is semigroup.

(ii) Again, for $\forall, a, b, c \in L$ and by definitions (2.1), (3.1), (3.2), (3.4) and (3.5) we have

$$ab \cdot c = cb \cdot a \Rightarrow a \cdot ab = c \cdot cb$$
$$aa \cdot b = c \cdot cb \Rightarrow aa \cdot cb = cb$$
$$(cb \cdot a)a = cb \Rightarrow (cb \cdot a)b = ca$$
$$ba \cdot cb = ca \Rightarrow bc \cdot ab = ca$$
$$a \cdot bc = ab \cdot c$$
$$\text{thus, } a(bc) = (ab)c.$$

Hence, L is semigroup.

Relation of Right Alternative, Right Nuclear square and Middle Nuclear Square LA-Semigroups

In this section we proved that right alternative LA-Semigroup L is neither right nor middle nuclear square LA-Semigroup. But the combination of any two of them gives the other. First we construct an example which show that neither of right alternative, right nuclear square nor middle nuclear square LA-Semigroup are containing in each other.

Example 5. *In the following Cayley's Table, we have*

*	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	2	2	3

Table(i)

*	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	2	1	3

(Table (ii))

*	1	2	3	4
1	1	1	1	4
2	1	1	1	4
3	1	2	2	4
4	4	4	4	1

(Table (iii))

Table(i) is right alternative LA-Semigroup which is neither Right Nuclear Square LA-Semigroup nor middle nuclear square LA-Semigroup
Table(ii) Right Nuclear Square LA-Semigroup, which is neither right alternative LA-Semigroup, nor middle nuclear square LA-Semigroup
Table(iii) is middle nuclear square LA-Semigroup, which is neither right alternative LA-Semigroup, nor Right nuclear square LA-Semigroup
Now we have the following result to prove that every right alternative LA-Semigroup satisfying is right nuclear square LA-Semigroup if and only if it is middle nuclear square LA-Semigroup.
Theorem 5. *Let L be a right alternative LA-Semigroup then A is right nuclear square LA-Semigroup property* $\Longleftrightarrow$ *L is middle nuclear square LA-Semigroup.*
Proof. Let L be right alternative LA-Semigroup satisfying the right nuclear square LA-Semigroup property, then we have to prove that it is middle nuclear square LA-Semigroup. for this let $\forall a,b,c \in A$, and by identities (3.1), (3.6) and (3.9) we have

$$a \cdot b^2c \quad = a(cb \cdot b) = a(cb^2) = ac \cdot b^2 = b^2c \cdot a = (cb \cdot b)a = (cb^2)a = ab^2 \cdot c$$

thus, $a \cdot b^2c \quad = \quad ab^2 \cdot c$.
Hence, L is middle nuclear square LA-semigroup.
Conversely: Let L be right alternative LA-semigroup satisfying the middle nuclear square LA-semigroup property, then we have to prove that A is right nuclear square LA-semigroup, for this let $\forall a,b,c \in A$, so by identities (3.1), (3.6) and (3.10), we have
$a \cdot bc^2 = a(bc \cdot c) = a(c^2b) = ac^2 \cdot b = bc^2 \cdot a = (bc \cdot c)a \; (c^2b)a = ab \cdot c^2$
Hence, L is right nuclear square LA-semigroup.

Relation of Right Alternative, Right Nuclear square and Jordan LA-semigroups

In this section we establish the relation among the right alternative, right nuclear square and Jordan LA-semigroups. First we prove by constructing an example that a right alternative LA-semigroup is neither right nuclear square LA-semigroup nor Jordan LA-semigroup. But the combination of right alternative and right nuclear square LA-semigroup becomes Jordan LA-semigroup and vice versa.
Example 6. *(a) Cayley's table(i) is Jordan LA-Semigroup, which neither Right Nuclear Square LA-semigroup nor right alternative LA-semigroup.*
Cayley's table(ii) Right Nuclear Square LA-semigroup, which is neither right alternative LA-semigroup nor Jordan LA-semigroup.
Cayley's table(iii) is right alternative LA-semigroup, which is neither right nuclear square LA-semigroup nor Jordan LA-semigroup.

*	1	2	3	4
1	1	2	2	2
2	2	1	1	1
3	2	1	1	1
4	3	1	1	1

(Table (i))

*	1	2	3	4	5
1	1	1	3	3	3
2	3	3	1	1	1
3	2	2	1	1	2
4	3	3	1	1	2
5	3	4	1	1	2

*	1	2	3	4
1	2	3	4	3
2	3	2	3	2
3	2	3	2	3
4	3	2	3	2

(Table (iii))

Now we have to prove the following result

Theorem 6. *Let L be a right alternative LA-Semigroup then A is right nuclear square LA-Semigroup ⇐⇒ L is Jordan LA-Semigroup.*

Proof. Let L be right alternative LA-Semigroup satisfying the right nuclear square LA-Semigroup property, then we have to prove that it is Jordan LA-Semigroup. For this let ∀,a,b,c ∈ L, and by (3.1), (3.6) and (3.9), we have

$a(b^2 \cdot c) = a(cb \cdot b) = a(cb^2) = ac \cdot b^2 = (ac \cdot b) \cdot b = b^2 \cdot ac.$

Hence, L is Jordan LA-semigroup.

Conversely: Let L be a right alternative LA-semigroup satisfying the Jordan LA-semigroup property, then we have to prove that it right nuclear square LA-semigroup. for this let ∀,a,b,c ∈ A and using the definitions (3.1), (3.6) and (3.11), we have

$ab \cdot c^2 = (ab \cdot c)c = c^2 \cdot ab = a \cdot c^2b = a \cdot (bc \cdot c) = a \cdot bc^2.$

Hence, L is right nuclear square LA-semigroup.

Relation of Alternative LA-semigroup and LA-Band with other subclasses

Now here in this section we find the relation of alternative LA Band with other subclasses of LA-semigroups, such as paramedial, middle nuclear square, self-dual, stein, LA*, Jordan and T^1-LA-Semigroups. Before finding the relation we construct an example by using GAP-4 to show that neither LA Band nor Alternative LA-semigroup related to these classes.

Example 7. *In the following Cayley's tables*
Cayley's table(i) is Alternative LA-semigroup, which is not Middle nuclear square LA-semigroup.
Cayley's table(ii) is Alternative LA-semigroup, which is not Paramedial LA-semigroup.
Cayley's table(iii) Right Alternative LA-semigroup, which is neither Self-dual nor Stein LA-semigroup.
Cayley's table(iv) Right Alternative LA-semigroup, which is neither LC nor RC-LA-semigroup.
Cayley's table(v) Right Alternative LA-semigroup, which is nor Jordan LA-semigroup.

*	1	2	3	4	5
1	3	3	4	4	4
2	4	5	4	4	2
3	4	4	4	4	4
4	4	4	4	4	4
5	4	2	4	4	5

(Table (i))

*	1	2	3	4
1	4	3	4	4
2	4	4	1	4
3	2	4	4	4
4	4	4	4	4

(Table (ii))

*	1	2	3	4	5
1	1	1	3	3	5
2	1	1	3	4	5
3	3	3	5	5	1
4	3	3	5	5	1
5	5	5	1	1	3

(Table (iii))

*	1	2	3	4	5
1	1	1	3	3	3
2	1	1	4	5	5
3	3	3	1	1	1
4	3	5	1	1	1
5	3	3	1	1	1

(Table (iv))

*	1	2	3	4
1	2	3	4	3
2	3	2	3	2
3	2	3	2	3
4	3	2	3	2

(Table (v))

Example 8. *The following Cayley's table (vi) is an LA-Band*

*	1	2	3	4	5
1	1	3	2	5	4
2	4	2	5	1	3
3	5	4	3	2	1
4	3	5	1	4	2
5	2	1	4	3	5

(Table (vi))

which is non of the following
Middle nuclear square LA-semigroup.
Paramedial LA-semigroup.
Self-dual nor Stein LA-semigroup.
LC nor RC-LA-semigroup.
Jordan LA-semigroup.
Now we prove the following result.

Theorem 7. *Let* L *be an alternative AG Band. Then*

(a) L *is paramedial LA-Semigroup.*
(b) L *is middle nuclear square LA-Semigroup.*
(c) L *is self-dual LA-Semigroup.*
(d) L *is stein LA-Semigroup.*
(e) L *is Jordan LA-Semigroup.*
(f) L *is T^1-LA-Semigroup.*
(g) L *is LA*-Semigroup.*
(h) L *is LC-LA-Semigroup.*
(i) L *is RC-LA-Semigroup.*

Proof. Let L be an alternative LA Band. Then

(a) $\forall a, b, c, d \in$ L, and by (2.1), (3.1), (3.6) and (3.7), we have

$$ab \cdot cd = ab \cdot (cc \cdot dd) = (a \cdot cc)(b \cdot dd) = (ac \cdot c)(bd \cdot d) = (cc \cdot a)(dd \cdot b)= (cc \cdot dd)(ab) = ((dd \cdot c)c)(aa \cdot b)$$
$$= (dd \cdot cc)(ba \cdot a)=(dc)(b \cdot aa)=db \cdot ca.$$

Hence, L is paremedial LA-semigroup.

(b) $\forall a,b,c \in$ L and by (2.1), (3.1), (3.6) and (3.7), we have

$$a \cdot b^2c = a(cb \cdot b)= a(cb^2 = aa \cdot cb^2 = ac \cdot ab^2 = (a \cdot cc)(ab^2)=(ac \cdot c)(ab^2) = (cc \cdot a)(ab^2) =(ab^2 \cdot a)(cc) =((aa \cdot b^2)a)c$$
$$=((b^2a \cdot a)a)c=((b^2 \cdot aa)a)c =(b^2a \cdot a)c =(a^2b^2)c = ab^2 \cdot c.$$

Hence, L is middle nuclear square LA-semigroup.

(c) $\forall a,b,c \in$ A, and (3.1), (3.6) and (3.7), we have

$$a \cdot bc = aa \cdot bc = (bc \cdot a)a = bc \cdot aa = (b \cdot cc)a = (bc \cdot c)a=(cc \cdot b)a = ab \cdot cc$$
$$= (aa \cdot b)(cc) = (ba \cdot a)cc=(b \cdot aa)cc=ba \cdot cc = (ba \cdot c)c = cc \cdot ba = c \cdot ba.$$

Hence, L is self-dual LA-semigroup.

(d) Again, $\forall a,b,c \in$ L and (3.1), (3.6) and (3.7), we have

$$a \cdot b^2c=a(cb \cdot b) = a(cb^2)= aa \cdot cb^2 =(cb^2 \cdot a)a =cb^2 \cdot aa =(cb \cdot b)(aa)= (b^2c)a = ac \cdot b^2 = (ac \cdot b)b = b^2 \cdot ac.$$

Hence, L is Jordan LA-semigroup.

(e) Similarly, $\forall, a, b, c \in$ L, and (2.1), (3.1), (3.6) and (3.7), we have

$$ab \cdot c = (a \cdot bb)c = (ab \cdot b)c = cb \cdot ab = ca \cdot bb = (c \cdot aa)bb = (ca \cdot a)(bb) = (aa \cdot c)bb = ac \cdot bb = (ac \cdot b)b = bb \cdot ac = b \cdot ac.$$

Hence, L is LA*-semigroup.

(f) Now $\forall, a, b, c \in$ L, and (3.1), (3.6) and (3.7) we have

$$ab \cdot c = (a \cdot bb)c = (ab \cdot b)c = (bb \cdot a)c = ba \cdot c.$$

Hence, L is LC-LA-semigroup.

(g) Again, $\forall, a, b, c \in$ L, (3.1), (3.6) and (3.7), we get

$$a \cdot bc = a(b \cdot cc) = a(bc \cdot c) = a(cc \cdot b) = a \cdot cb.$$

Hence, L is RC-LA-semigroup.

(h) Similarly, $\forall, a, b, c \in$ L and using (3.1), (3.6) and (3.7), we have

$$bc \cdot a = bc \cdot aa = (bc \cdot a)a = aa \cdot bc = a \cdot bc.$$

Hence, L is Stein LA-semigroup.

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