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D-NEIGHBORHOOD POLYNOMIAL AND DEGREE BASED TOPOLOGICAL INDICES OF INDU-BALA PRODUCT OF TWO CYCLES. A NEW APPROACH

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Abstract
A deep studies shows that indu-bala product which is a graphical presentation of product of two graphs was presented by Indulal and Balakrishnan (2016) [1]. The role of d-neighborhood polynomial cannot be denied for the express purpose of determining physical, chemical, biological characteristics of Indu-bala product of two graphs which is to be examined provided that neighborhood degree sum based indices help the d-neighborhood polynomial. This paper introduces a new approach in finding the topological indices of certain graphs by considering of d-neighborhood degree based polynomial that has been derived from Indu-bala product of two cycle.

1 Introduction

The consideration of the graphs interconnected without the help of any loop or edge related to the main concern or topic of this article. Finding a cause of any disease has always been a concern for the persons involved in drug discovery. No wonder drug discovery is a very wide sphere that surrounds so many things like the validity of the target, identification and optimization of factors involved [2]. As far as identification of any symptoms or lead is concerned, the condition of different compounds and there origin is examined as a recent prerequisite that advanced pharmacology has suggested. This leads the concerned to recommend the most appropriate clinically used medicine [3, 4]. When it comes to optimizing the leads, the chemical alteration of molecules or performed under the set rules of pharmacology in order to make the medicines usable under any circumstances [5]. These two processes do not take us a way from keeping in view the properties of drug as well as the condition and various functions of human body for warding off the illness [6, 7]. In addition heretical computation of molecules is carried keeping in view their various biological activities for their ultimate influence on disease. Likewise different parameters and gauges of these drug molecules are also evaluated for their electronic features. The quantity structure activity relationships basis exclusively on molecular indicators which bring forth several experimental and theoretical results [8]. Molecular indicators that are based on indices of topology are believed to have a very significant role qualitative structure activity relationships studies. As these indices are a clear manifestation of those numerical values that are very much associated with chemical compositions. The reason for their strong influence is that they are represented by authentic methods of topology and further different indicators determines their structures-hydrophobicity constants [9]. The methods described earlier have given to molecular topology that has been successful in established itself for the true analysis of chemicals for their physical and biological properties. The basics of molecular topology are used to represent chemicals graphically. [10-12].

Consider $V(\chi)$ and $E(\chi)$ be the set of vertices and links of graph χ. The degree of a vertex $u \in V(\chi)$ denoted by d_u, is the number of edges incident with u. S_u denotes the degree sum of neighbors of u in χ . Adjacent nodes are called neighbors. Graph theory, basically deals with the compounds and molecules on the basis of their qualities and functionalities. we resort to quantitative structure property relationships // quantitative structure activity relationships (QSPR/QSAR) in order to determine structure, physico-chemical property and biological features of compounds. This helps us determines multidimensional features and activities of molecules as well. Resultantly this method is significantly helpful in the process for discovering drugs and anticipating their features and functions of molecules. This method as it involves topology eliminates the requirements of a wet lab at all. Indices in the field of topology, that a destined to real numbers in the form of graph are used in QSPR/QSAR analysis. There is no doubt in it that topological descriptors are vary hard to defined as they lead to many other terminologies or things that are closely related to them. Inspite of the complexity of definitions they are unable to bring any change in the graphs that a isomorphic in nature.

The main reason why M-polynomial was introduced is because it saves a lot of effort in computing the topological indices. So far, much of recent research has been focused upon calculating closed formulae for topological indices for groups of graphs. For the undermining already obtained, integration, differentiation and any kind of indices of topology many polynomial of algebra have been brought into use [13-15]. Among these polynomials the M-Polynomial proved to be giving the reacquired results by given berth to degree based topological indices [16, 17]. With the pace of researches being carried the introduction of new indices is no exception [18-25]. To make the reason for the discovery of new indices is M-polynomial being the easiest method for determining neighborhood M-polynomial and analogous for the proper functioning of the former for degree base indices. Due to the complexities in bringing forth degree based indices the researchers emphasized on the use of different indices of topology to establish the structures of molecules[26-28]. Researchers have proved that the study of topological indices has been a pivotal importance in establishing boiling point, surface tension, molar fraction and vaporization etc. The importance of topological indices can also be determined by the description of different biological behavior like nutritively, toxicity, and stimulation found in the growth of cell etc. In short chemical structure cannot be computed without the significant topological indicator[29-31]. The article should help very much understand the calculation , evaluation and computation of multidimensional indicators of topology that are based on d-neighborhood polynomial for a clear-cut conviction about indu-bala product of two cycle.

2 Preliminaries

It was the year 1972 when the world first came to know the degree based topological indices by Gutman and Trinajstic. Before this coinage the world knew it only the name of zagreb index. In 1975,the recent times researchers and authors as Randic index as he is the man who worked on their branching and further excution of related things. The degree based indices of end nodes of edges for a graph χ are defined as follows:$I(\chi) = \Sigma_{uv\in E(\chi)F(d_u,d_y)}$,where $F(d_u, d_y)$ is defined for different well-established descriptors. For more degree based indices, readers are referred to [32-34]. The M-polynomial of a graph χ is defined as,

$$M(\chi, x, y) = \sum_{i\leq j} m_{i,j} x^i y^j \quad (1)$$

Where $m_{i,j}$ is the total number of edges $uv \in E(\chi)$ such that $\{d_u, d_v\} = \{i, j\}$. The d-neighborhood polynomial of a graph χ is defined as:

$$d_N M(\chi, x, y) = \sum_{i\leq j} \varrho_{i,j} x^i y^j \quad (2)$$

Where $\varrho_{\{i,j\}}$ is the total number of edges $uv \in E(\chi)$ such that $\{S_u, S_v\} = \{i, j\}$. We use $d_N M(\chi)$ for $d_N M(\chi, x, y)$.

Definition 1 *[35] The Indubala product $C_m \blacktriangledown C_n$ at graphs C_m and C_n is obtain from two disjoint copies of join $C_m \vee C_n$ of C_m and C_n by connecting. The corresponding vertices in the two copies of C_n. The cardinality of vertices and edges in $C_m \blacktriangledown C_n$ is given by $|V(C_m \blacktriangledown C_n)| = 2(m + n)$ and $|E(C_m \blacktriangledown C_n)| = 2(m + n + mn + 1)$.*

Definition 2 *Let u be a vertex of the graph χ. Then the sum of the degrees of vertex u and sum of the all the degrees of neighbors which are adjacent to the vertex u in χ is called d-neighborhood of vertex u. The d-neighborhood of vertex $u \in \chi$ is denoted by*

$DN_u = d_u + \sum_{uv\in E(\chi)} d_v, \forall\ v \in V(\chi)$. *For every v adjacent to u.*

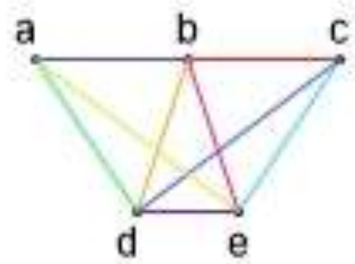


Figure 1: Simple graph with vertex degree

In Figure (1) we have, $d_a = d_c = 3, d_b = d_d = d_e = 4$, and $S_a = S_c = 15, S_b = S_d = S_e = 18$. Thus we have the followings values $m_{\{3,4\}} = 6, m_{\{4,4\}} = 3, \varrho_{\{15,18\}} = 6, \varrho_{\{18,18\}} = 3$

We have the following equations.

$$M(\chi, x, y) = 6x^3y^4 + 3x^4y^4 \quad (3)$$

$$d_N M(\chi, x, y) = 6x^{15}y^{18} + 3x^{18}y^{18} \quad (4)$$

Formulation of topological indices using d-Neighborhood polynomial for a graph χ.

d-neighborhood Third Version of First Zagreb Index

$$M_1'(\chi) = (D_x + D_y)(d_N M(\chi)\ \ at\ \ x = y = 1 \quad (5)$$

d-neighborhood Second Zagreb Index [20]

$$M_2^*(\chi) = (D_x D_y)(d_N M(\chi))\ \ at\ \ x = y = 1 \quad (6)$$

, d-neighborhood Forgotten Topological Index

$$F_N^*(\chi) = (D_x^2 + D_y^2)(d_N M(\chi)\ \ at\ \ x = y = 1 \quad (7)$$

In [21] d-neighborhood Second Modified Zagreb Index

$$^{NM}M_2(\chi) = (S_x S_y)(d_N M(\chi)\ \ at\ \ x = y = 1 \quad (8)$$

In d-Neighborhood General Randic Index

$$NR_\alpha(\chi) = (D_x^\alpha D_y^\alpha)(d_N M(\chi)\ \ at\ \ x = y = 1 \quad (9)$$

In [21] d-neighborhood Third NDe Index

$$d_N D_3(\chi) = D_x D_y (D_x + D_y)(d_N M(\chi))\ \ at\ \ x = y = 1 \quad (10)$$

In [21] d-neighborhood [1ex] Fifth NDe Index

$$d_N D_5(\chi) = (D_x S_y + S_x D_y)(d_N M(\chi)\ \ at\ \ x = y = 1 \quad (11)$$

In [21] d-neighborhood Harmonic Index

$$d_N H(\chi) = 2S_x J(d_N M(\chi) \ at \ x = 1 \tag{12}$$

. In [21] d-neighborhood Inverse Sum Index

$$d_N IN(\chi) = S_x J D_x D_y (d_N M(\chi)) \ at \ x = 1 \tag{13}$$

In [21]d-neighborhood Sanskruti Index

$$d_N S(\chi) = S_x^3 J D_x^3 D_y^3 (d_N M(\chi)) \ at \ x = 1. \tag{14}$$

$$D_u = u\frac{\partial(f(u,v))}{\partial u}, D_v = v\frac{\partial(f(u,v))}{\partial v}, S_u = \int \frac{f(u,v)}{u}du, S_v = \int \frac{f(u,v)}{v}dv, \tag{15}$$

$$J(h(u,v)) = f(u,u), Q_\alpha(h(u,v)) = u^\alpha h(u,v). \tag{16}$$

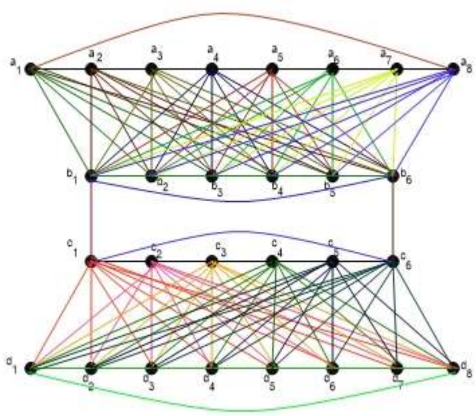
3 Computational Results on d-Neighborhood Polynomial for Topological Descriptors for Indu-Bala Product of Two Cycles C_m and C_n for all m, n where as $m \leqslant n, m \geqslant n$

Our basic theme of studying d-Neighborhood Polynomial and its related all descriptors of Indu-bala product of two cycle graphs see Figure 2 and 3.

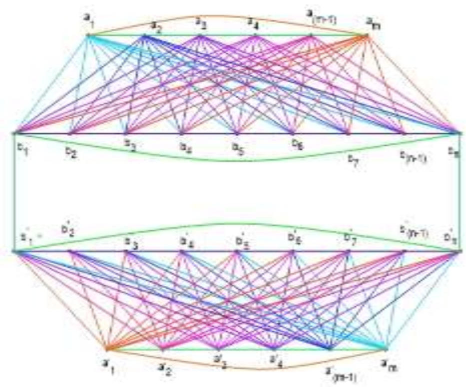
3.1 Results

We distinguish the parts of graph is known as vertices and links or Edge and also the partitions of the edge of Indu-Bala product of cycle two paths.Total number of vertices and edges of Indu-bala product of two cycle graphs are $|V(\chi)| = 2(m+n)$ and $|E(\chi)| = 2(mn+m+n+1)$. The partition of vertices with respect to their degree is denoted by a,b,c and d is given as follows.

$$a = mn + 5n + 8, b = mn + 5m + 6, c = mn + 5m + 7, d = mn + 6m + 11. \tag{17}$$



(a)



(b)

Figure 2: (a) Indu-Bala graph $C_8 \blacktriangledown C_6$ (b) Indu-Bala graph $C_m \blacktriangledown C_n$

Theorem 1 *Let χ be a Indu-Bala graph $C_m \blacktriangledown C_n$, where $\forall m, n \geq 3$ .We have*

$$\begin{aligned} d_N M(\chi; x, y) = & (2m)x^{\{mn+5n+8\}}y^{\{mn+5n+8\}} + m(2n-8)x^{\{mn+5n+8\}}y^{\{mn+5m+6\}} \\ & +4mx^{\{mn+5n+8\}}y^{\{mn+5m+7\}} + 4mx^{\{mn+5n+8\}}y^{\{mn+6m+11\}} \\ & +2(n-5)x^{\{mn+5m+6\}}y^{\{mn+5m+6\}} + 4x^{\{mn+5m+6\}}y^{\{mn+5m+7\}} \\ & +4x^{\{mn+5m+7\}}y^{\{mn+6m+11\}} + 4x^{\{mn+6m+11\}}y^{\{mn+6m+11\}}. \end{aligned}$$

Proof 1 *In Figure (2) and (3) , we will compute d-Neighborhood Polynomial. Partitions of Edge set of χ of two cycle graph are as follows.*

$$|E_{(a,a)}| = |uv \in E(\chi); \varrho_u = \varrho_v = a| = 2m;\ |E_{(a,b)}| = |uv \in E(\chi); \varrho_u = a, \varrho_v = b| = m(2n-8);$$
$$|E_{(a,c)}| = |uv \in E(\chi); \varrho_u = a, \varrho_v = c| = 4m;\ |E_{(a,d)}| = |uv \in E(\chi); \varrho_u = a, \varrho_v = d| = 4m;$$
$$|E_{(b,b)}| = |uv \in E(\chi); \varrho_u = b, \varrho_v = b| = 2(n-5);\ |E_{(b,c)}| = |uv \in E(\chi); \varrho_u = b, \varrho_v = c| = 4;$$
$$|E_{(c,d)}| = |uv \in E(\chi); \varrho_u = c, \varrho_v = d| = 4;\ |E_{(d,d)}| = |uv \in E(\chi); \varrho_u = \varrho_v = d| = 4.$$

$$d_N M(\chi; x, y) = \sum_{i \le j} \varrho_{\{i,j\}}(\chi, x^i y^j) = \sum_{a \le a} \varrho_{\{a,a\}}(\chi) x^a y^a + \sum_{a \le b} \varrho_{\{a,b\}}(\chi) x^a y^b +$$
$$\sum_{a \le c} \varrho_{\{a,c\}}(\chi) x^a y^c + \sum_{a \le d} \varrho_{\{a,d\}}(\chi) x^a y^d + \sum_{b \le b} \varrho_{\{b,b\}}(\chi) x^b y^b + \sum_{b \le c} \varrho_{\{b,c\}}(\chi) x^b y^c +$$
$$\sum_{c \le d} \varrho_{\{c,d\}}(\chi) x^c y^d + \sum_{d \le d} \varrho_{\{d,d\}}(\chi) x^d y^d$$
$$= \sum_{uv \in E_{\{a,a\}}} \varrho_{\{a,a\}}(\chi) x^a y^a + \sum_{uv \in E_{\{a,b\}}} \varrho_{\{a,b\}}(\chi) x^a y^b + \sum_{uv \in E_{\{a,c\}}} \varrho_{\{a,c\}}(\chi) x^a y^c$$
$$+ \sum_{uv \in E_{\{a,d\}}} \varrho_{\{a,d\}}(\chi) x^a y^d + \sum_{uv \in E_{\{b,b\}}} \varrho_{\{b,b\}}(\chi) x^b y^b + \sum_{uv \in E_{\{b,c\}}} \varrho_{\{b,c\}}(\chi) x^b y^c$$
$$+ \sum_{uv \in E_{\{c,d\}}} \varrho_{\{c,d\}}(\chi) x^c y^d + \sum_{uv \in E_{\{d,d\}}} \varrho_{\{d,d\}}(\chi) x^d y^d$$
$$= |E_{\{a,a\}}| x^a y^a + |E_{\{a,b\}}| x^a y^b + |E_{\{a,c\}}| x^a y^c + |E_{\{a,d\}}| x^a y^d$$
$$+ |E_{\{b,b\}}| x^b y^b + |E_{\{b,c\}}| x^b y^c + |E_{\{c,d\}}| x^c y^d + |E_{\{d,d\}}| x^d y^d$$
$$= (2m) x^{\{mn+5n+8\}} y^{\{mn+5n+8\}} + m(2n-8) x^{\{mn+5n+8\}} y^{\{mn+5m+6\}}$$
$$+ 4m x^{\{mn+5n+8\}} y^{\{mn+5m+7\}} + 4m x^{\{mn+5n+8\}} y^{\{mn+6m+11\}}$$
$$+ 2(n-5) x^{\{mn+5m+6\}} y^{\{mn+5m+6\}} + 4 x^{\{mn+5m+6\}} y^{\{mn+5m+7\}}$$
$$+ 4 x^{\{mn+5m+7\}} y^{\{mn+6m+11\}} + 4 x^{\{mn+6m+11\}} y^{\{mn+6m+11\}}.$$

Theorem 2 *In Indu-Bala graph $C_m \blacktriangledown C_n$, where $\forall m, n \ge 3$ .Then Third version of d-Neighborhood Zagreb index $dM_1^*(\chi)$ is*

$$dM_1'(\chi) = mn(2mn + 6m + 8n + 11) + m(4m + 94) + 14n + 116.$$

Proof 2 *We use the value of d-neighborhood polynomial and we calculate the third version of d-Neighborhood First Zagreb index is*

$$d_N M(\chi; x, y) = 2m x^a y^a + m(2n-8) x^a y^b + 4m x^a y^c + 4m x^a y^d + 2(n-5) x^b y^b + 4 x^b y^c + 4 x^c y^d + 4 x^d y^d$$

$$d_N M(\chi; x, y) = f(x, y); \quad D_x f(x, y) = x \frac{\partial f}{\partial x}; \quad (D_x + D_y) f(x, y)$$
$$= 2ma(x^{a-1} y^a + x^a y^{a-1}) + m(2n-8)(a x^{a-1} y^b + b x^a y^{b-1}) + 4m(a x^{a-1} y^c + c x^a y^{c-1})$$
$$+ 4m(a x^{a-1} y^d + d x^a y^{d-1}) + 2(n-5) b(x^{b-1} y^b + x^b y^{b-1}) + 4(a x^{b-1} y^c + c x^b y^{c-1})$$
$$+ 4(c x^{c-1} y^d + d x^c y^{d-1}) + 4d(x^{d-1} y^d + x^d y^{d-1})$$
$$M_1'(\chi) = (D_x + D_y)(f(x, y)|_{(x=y=1)} = mn(2mn + 6m + 8n + 11) + m(4m + 94) + 14n + 116.$$

Theorem 3 *In Indu-Bala graph $C_m \blacktriangledown C_n$, where $\forall m, n \ge 3$ .Then d-Neighborhood second Zagreb index $dM_2^*(\chi)$ is*

$$dM_2'(\chi) = m^3(2n^3 + 12n^2 + 4n) + m^2(12n^3 + 70n^2 + 463n + 146)$$
$$+ m(50n^3 - 140n^2 + 44n + 1280) + 72n + 216.$$

Proof 3 *We use the value of d-neighborhood polynomial and calculate the value of Second Zagreb index is*

$$d_N M(\chi; x, y) = 2m x^a y^a + m(2n-8) x^a y^b + 4m x^a y^c + 4m x^a y^d + 2(n-5) x^b y^b + 4 x^b y^c + 4 x^c y^d + 4 x^d y^d$$

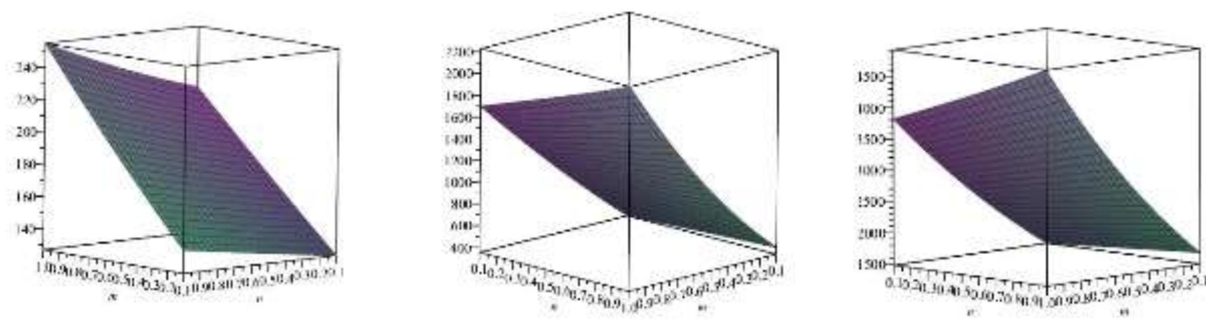
$$D_x D_y f(x, y) = 2ma^2 x^{a-1} y^{a-1} + m(2n-8) ab x^{a-1} y^{b-1} + 4mac x^{a-1} y^{c-1} + 4mad x^{a-1} y^{d-1}$$
$$+ 2(n-5) b^2 x^{b-1} y^{b-1} + 4bc x^{b-1} y^{c-1} + 4cd x^{c-1} y^{d-1} + 4d^2 x^{d-1} y^{d-1}$$
$$dM_2'(\chi) = D_x D_y f(x, y)|_{(x=y=1)}$$
$$= 2ma^2 + m(2n-8)ab + 4mac + 4mad + 2(n-5)b^2 + 4bc + 4cd + 4d^2$$
$$dM_2'(\chi) = m^3(2n^3 + 12n^2 + 4n) + m^2(12n^3 + 70n^2 + 463n + 146)$$
$$+ m(50n^3 - 140n^2 + 44n + 1280) + 72n + 216.$$

Theorem 4 *In Indu-Bala graph* $C_m \blacktriangledown C_n$, *where* $\forall m, n \geq 3$ *.Then d-Neighborhood forgotten topological index* $dF_N^*(\chi)$ *is*

$$dF_N^*(\chi) = m^3(4n^3 + 20n^2 + 58n + 44) + m^2(24n^3 + 140n^2 + 282n + 560)$$
$$+m(50n^3 + 160n^2 + 656n + 1832) + 144n + 1268.$$

Proof 4 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood forgotten topological index is*

$$d_N M(\chi; x, y) = 2mx^a y^a + m(2n-8)x^a y^b + 4mx^a y^c + 4mx^a y^d + 2(n-5)x^b y^b + 4x^b y^c$$
$$+4x^c y^d + 4x^d y^d$$
$$(D_x^2 + D_y^2)f(x, y) = 4ma^2 x^a y^a + m(2n-8)(a^2 + b^2)x^a y^b + 4m(a^2 + c^2)x^a y^c$$
$$= 4m(a^2 + d^2)x^a y^d + 4(n-5)b^2 x^b y^b + 4(b^2 + c^2)x^b y^c + 4(c^2 + d^2)x^c y^d + 8d^2 x^d y^d$$
$$F_N^*(\chi) = (D_x^2 + D_y^2)f(x, y)|_{(x=y=1)} = 4ma^2 + m(2n-8)(a^2 + b^2) + 4m(a^2 + c^2)$$
$$= 4m(a^2 + d^2) + 4(n-5)b^2 + 4(b^2 + c^2) + 4(c^2 + d^2) + 8d^2$$
$$F_N^*(\chi) = m^3(4n^3 + 20n^2 + 58n + 44) + m^2(24n^3 + 140n^2 + 282n + 560)$$
$$+m(50n^3 + 160n^2 + 656n + 1832) + 144n + 1268.$$



(a) (b) (c)

Figure 3: (a) d-Nhbd 3rd version of Zagreb Index for all m, n; (b) d-Nhbd Second Zagreb Index for all m, n; (c) d-Nhbd Forgotten Topological Index for all m, n

Theorem 5 *In Indu-Bala graph* $C_m \blacktriangledown C_n$,*where* $\forall m, n \geq 3$ *.Then d-Neighborhood general Randic index* $d_N R_\alpha(\chi)$ *is*

$$d_N R_\alpha(\chi) = 2m(m^2n^2 + 25n^2 + 64 + 10mn^2 + 80n + 16mn)^\alpha + m(2n-8)(m^2n^2 + 5m^2n$$
$$+39mn + 5mn^2 + 40m + 30n + 48)^\alpha + 4m(m^2n^2 + 5m^2n + 40mn + 5mn^2$$
$$+35n + 40m + 56)^\alpha + 4m(m^2n^2 + 6m^2n + 49mn + 5mn^2 + 48m + 55n + 88)^\alpha$$
$$+2(n-5)(m^2n^2 + 25m^2 + 36 + 10m^2n + 60m + 12mn)^\alpha + 4(m^2n^2 + 25m^2$$
$$+10m^2n + 52mn + 260m + 168)^\alpha + 4(m^2n^2 + 11m^2n + 30m^2 + 18mn + 97m + 77)^\alpha$$
$$+4(m^2n^2 + 36m^2 + 12m^2n + 132m + 22mn + 121)^\alpha.$$

Proof 5 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood general Randic index is*

$d_N M(\chi; x, y) = 2mx^a y^a + m(2n-8)x^a y^b + 4mx^a y^c + 4mx^a y^d + 2(n-5)x^b y^b + 4x^b y^c + 4x^c y^d + 4x^d y^d$; $\quad d_N R_\alpha(\chi) = D_x^\alpha D_y^\alpha f(x, y)|_{(x=y=1)}$

$$D_y^\alpha f(x, y) = 2m(a)^\alpha x^a y^a + m(2n-8)(b)^\alpha x^a y^b + 4m(c)^\alpha x^a y^c + 4m(d)^\alpha x^a y^d$$
$$+2(n-5)(b)^\alpha x^b y^b + 4(c)^\alpha x^b y^c + 4(d)^\alpha x^c y^d + 4(d)^\alpha x^d y^d$$
$$D_x^\alpha D_y^\alpha f(x, y) = 2m(a)^{2\alpha} x^a y^a + m(2n-8)(ab)^\alpha x^a y^b + 4m(ac)^\alpha x^a y^c + 4m(ad)^\alpha x^a y^d$$
$$+2(n-5)(b)^{2\alpha} x^b y^b + 4(bc)^\alpha x^b y^c + 4(cd)^\alpha x^c y^d + 4(d)^{2\alpha} x^d y^d$$

$$d_NR_\alpha(\chi) = D_x^\alpha D_y^\alpha f(x,y)|_{(x=y=1)} = 2m(a)^{2\alpha} + m(2n-8)(ab)^\alpha + 4m(ac)^\alpha + 4m(ad)^\alpha + 2(n-5)(b)^{2\alpha} + 4(bc)^\alpha + 4(cd)^\alpha + 4(d)^{2\alpha}$$

$$d_NR_\alpha(\chi) = 2m(m^2n^2 + 25n^2 + 64 + 10mn^2 + 80n + 16mn)^\alpha + m(2n-8)(m^2n^2 + 5m^2n + 39mn + 5mn^2 + 40m + 30n + 48)^\alpha + 4m(m^2n^2 + 5m^2n + 40mn + 5mn^2 + 35n + 40m + 56)^\alpha + 4m(m^2n^2 + 6m^2n + 49mn + 5mn^2 + 48m + 55n + 88)^\alpha + 2(n-5)(m^2n^2 + 25m^2 + 36 + 10m^2n + 60m + 12mn)^\alpha + 4(m^2n^2 + 25m^2 + 10m^2n + 52mn + 260m + 168)^\alpha + 4(m^2n^2 + 11m^2n + 30m^2 + 18mn + 97m + 77)^\alpha + 4(m^2n^2 + 36m^2 + 12m^2n + 132m + 22mn + 121)^\alpha$$

Theorem 6 *In Indu-Bala graph $C_m \blacktriangledown C_n$, where $\forall m,n \geq 3$ .Then d-Neighborhood Third NDe index $d_ND_3(\chi)$ is*

$$d_ND_3(\chi) = m^4(4n^5 + 36n^4 + 88n^3 + 72n^2 + 16n) + m^3(40n^5 + 330n^4 + 2874n^3 + 3114n^2 + 3104n + 584) + m^2(-2n^4 + 5532n^3 + 13030n^2 + 75552n + 138120) + m(700n^4 + 4560n^3 - 15624n^2 + 32168n + 168784) + 1008n^2 + 8352n + 28080.$$

Proof 6 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood Third NDe index is*

$$d_NM(\chi;x,y) = 2mx^ay^a + m(2n-8)x^ay^b + 4mx^ay^c + 4mx^ay^d + 2(n-5)x^by^b + 4x^by^c + 4x^cy^d + 4x^dy^d$$

$$d_ND_3(\chi) = D_xD_y(D_x+D_y)f(x,y)|_{(a=b=1)}; \quad D_xD_y(D_x+D_y)f(x,y)$$

$$= (2ma^2x^{a-1}y^{a-1} + m(2n-8)abx^{a-1}y^{b-1} + 4macx^{a-1}y^{c-1} + 4madx^{a-1}y^{d-1} + 2(n-5)b^2x^{b-1}y^{b-1} + 4bcx^{b-1}y^{c-1} + 4cdx^{c-1}y^{d-1} + 4d^2x^{d-1}y^{d-1})(2ma(x^{a-1}y^a + x^ay^{a-1}) + m(2n-8)(ax^{a-1}y^b + bx^ay^{b-1}) + 4m(ax^{a-1}y^c + cx^ay^{c-1}) + 4m(ax^{a-1}y^d + dx^ay^{d-1}) + 2(n-5)b(x^{b-1}y^b + x^by^{b-1}) + 4(ax^{b-1}y^c + cx^by^{c-1}) + 4(cx^{c-1}y^d + dx^cy^{d-1}) + 4d(x^{d-1}y^d + x^dy^{d-1})); \quad D_xD_y(D_x+D_y)f(x,y)|_{(x=y=1)}$$

$$= (2ma^2 + m(2n-8)ab + 4mac + 4mad + 2(n-5)b^2 + 4bc + 4cd + 4d^2)(2ma + m(2n-8)(a+b) + 4m(a+c) + 4m(a+d) + 2(n-5)b + 4(a+c) + 4(c+d) + 4d)$$

$$d_ND_3(\chi) = (m^3(2n^3 + 12n^2 + 4n) + m^2(12n^3 + 70n^2 + 463n + 146) + m(50n^3 - 140n^2 + 44n + 1280) + 72n + 216)(mn(2mn + 6m + 8n + 11) + m(4m + 94) + 14n + 116).$$

$$d_ND_3(\chi) = m^4(4n^5 + 36n^4 + 88n^3 + 72n^2 + 16n) + m^3(40n^5 + 330n^4 + 2874n^3 + 3114n^2 + 3104n + 584) + m^2(-2n^4 + 5532n^3 + 13030n^2 + 75552n + 138120) + m(700n^4 + 4560n^3 - 15624n^2 + 32168n + 168784) + 1008n^2 + 8352n + 28080.$$

Theorem 7 *In Indu-Bala graph $C_m \blacktriangledown C_n$, where $\forall m,n \geq 3$ .Then d-Neighborhood [1ex] Fifth NDe index $d_ND_5(\chi)$ is*

$$d_ND_5(\chi) = \frac{1}{u}\{m^5(4n^5 + 68n^4 + 410n^3 + 1314n^2 + 3754n + 600) + m^4(24n^5 + 136n^4 + 2944n^3 + 11660n^2 + 54718n) + m^3(70n^5 + 3242n^4 + 5432n^3 + 52168n^2 - 10738n + 25428) + m^2(1056n^4 + 10630n^3 + 33598n^2 - 36008n - 17568) + m(1694n^3 + 26752n^2 + 5944n + 106928) + 9240n^2 + 42502n - 44352\};$$

$$where, u = m^4(n^4 + 16n^3 + 30n^2 + 150n) + m^3(5n^4 + 112n^3 + 1220n^2 + 2825n + 1200) + m^2(120n^3 + 1642n^2 + 9379n + 5320) + m(385n^2 + 8916n + 7736) + 2310n + 3696$$

Proof 7 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood [1ex] Fifth NDe index is*

$$d_NM(\chi;x,y) = 2mx^ay^a + m(2n-8)x^ay^b + 4mx^ay^c + 4mx^ay^d + 2(n-5)x^by^b + 4x^by^c + 4x^cy^d + 4x^dy^d.; \quad d_ND_5(\chi) = (D_xS_y + S_xD_y)f(x,y)|_{(x=y=1)}$$

$$D_xS_yf(x,y) = 2mx^ay^a + m(2n-8)\frac{a}{b}x^ay^b + 4m\frac{a}{c}x^ay^c + 4m\frac{a}{d}x^ay^d + 2(n-5)x^by^b + 4\frac{b}{c}x^by^c + 4\frac{c}{d}x^cy^d + 4x^dy^d$$

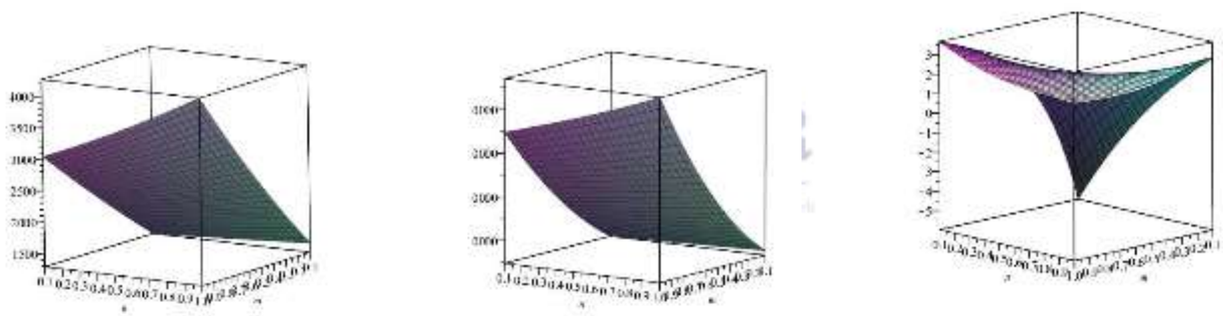
$$S_xD_yf(x,y) = 2mx^ay^a + m(2n-8)\frac{b}{a}x^ay^b + 4m\frac{c}{a}x^ay^c + 4m\frac{d}{a}x^ay^d + 2(n-5)x^by^b + 4\frac{c}{b}x^by^c + 4\frac{d}{c}x^cy^d + 4x^dy^d.;$$

$$(D_xS_y + S_xD_y)f(x,y) = 4mx^ay^a + m(2n-8)(\frac{b}{a}+\frac{b}{a})x^ay^b + 4m(\frac{c}{a}+\frac{a}{c})x^ay^c + 4m(\frac{d}{a}+\frac{a}{d})x^ay^d + 2(n-5)x^by^b + 4(\frac{c}{b}+\frac{b}{c})x^by^c + 4(\frac{d}{c}+\frac{c}{d})x^cy^d + 4x^dy^d;$$

$$(D_xS_y + S_xD_y)|_{(x=y=1)} = 4m + m(2n-8)(\frac{a^2+b^2}{ab}) + 4m(\frac{a^2+c^2}{ac}) + 4m(\frac{a^2+d^2}{ad}) + 4(n-5) + 4(\frac{b^2+c^2}{bc}) + 4(\frac{c^2+d^2}{cd}) + 8$$

$$= \frac{1}{abcd}(4mabcd + m(2n-8)cd(a^2+b^2) + 4mbd(a^2+c^2) + 4mbc(a^2+d^2) + 4(n-5)(abcd) + 4ad(b^2+c^2) + 4ab(c^2+d^2) + 8abcd)$$

$$= \frac{1}{u}\{m^5(4n^5 + 68n^4 + 410n^3 + 1314n^2 + 3754n + 600) + m^4(24n^5 + 136n^4 + 2944n^3 + 11660n^2 + 54718n) + m^3(70n^5 + 3242n^4 + 5432n^3 + 52168n^2 - 10738n + 25428) + m^2(1056n^4 + 10630n^3 + 33598n^2 - 36008n - 17568) + m(1694n^3 + 26752n^2 + 5944n + 106928) + 9240n^2 + 42502n - 44352\}.$$



(a) (b) (c)

Figure 4: (a) d-Nieghbrhood General Randic Index for all $\boldsymbol{m}, \boldsymbol{n}$ and $\boldsymbol{\alpha = 1}$; (b) d-Neighbuhood third $\boldsymbol{ND_e}$ Index for all $\boldsymbol{m}, \boldsymbol{n}$; (c) d-Neighbuhood Fifth $\boldsymbol{ND_e}$ Index for all $\boldsymbol{m}, \boldsymbol{n}$

Theorem 8 *In Indu-Bala graph* $C_m \blacktriangledown C_n$ *,where* $\forall m,n \geq 3$ *.Then d-Neighborhood Harmonic index* $d_NH(\chi)$ *is;*

$$d_NH(\chi) = \frac{1}{u}(m(2566822080 + 3833074924n + 2932551792n^2 + 1275066580n^3 + 318570084n^4 + 36104968n^5 + 3643164n^6 + 113288n^7) + m^2(5021045080 + 7920335216n + 6229404914n^2 + 2652419658n^3 + 615163682n^4 + 75460638n^5 + 4981764n^6 + 182000n^7 + 4704n^8) + m^3(5419378910 + 9213534072n + 6447240428n^2 + 2563585848n^3 + 519389002n^4 + 58160908n^5 + 4643528n^6 + 112448n^7) + m^4(3476853440 + 6446647114n + 4638363756n^2 + 1405778888n^3 + 256790698n^4 + 24650392n^5 + 1065816n^6) + m^5(1330973310 + 2712052284n + 1590787118n^2 + 475880176n^3 + 76067132n^4 + 4942744n^5) + m^6(288096900 + 651940590n + 349071702n^2 + 98334200n^3 + 12439288n^4) + m^7(30381000 + 78643800n + 45175700n^2 + 9883800n^3) + m^8(990000 + 3315000n + 3178400n^2) + 560893680 + 816357240n + 507891960n^2 + 144445800n^3 + 15027000n^4$$

$$where, u = abd(a+b)(a+c)(a+d)(b+c)(c+d)$$
$$= m(1856236320 + 3278958924n + 227227212n^2 + 998611036n^3 + 272379088n^4 + 49635936n^5 + 5588408n^6 + 193214n^7 + 8232n^8) + m^2(2928917040 + 5741936458n + 6891432945n^2 + 2139768366n^3 + 565462815n^4 + 84497222n^5 + 6458550n^6 + 337554n^7) + m^3(2508638680 + 5438153581 + 4654418900n^2 + 1998937210n^3 + 444106995n^4 + 47210194n^5 + 1842960n^6) + m^4(125915588 + 3026609932n + 2524685270n^2 + 931658030n^3 + 151960130n^4 + 8649048n^5) + m^5(370375600 + 1001661115n + 758770835n^2 + 211190210n^3 + 18895620n^4) + m^6(59148000 + 188054850n + 116686500n^2 + 18363900n^3) + m^7(3960000 + 17320500n + 6991500n^2) + 495000m^9n + 492972480 + 778223160n + 442084500n^2 + 108108000n^3 + 9652500n^4)$$

Proof 8 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood Harmonic index is*

$$d_NM(\chi;x,y) = 2mx^ay^a + m(2n-8)x^ay^b + 4mx^ay^c + 4mx^ay^d + 2(n-5)x^by^b + 4x^by^c + 4x^cy^d + 4x^dy^d;\ \ d_NH(\chi) = 2S_xjd_NM(\chi;x,y)(x,y)atx = 1;\ \ Jd_NM(\chi;x,y)$$
$$= 2mx^{2a} + m(2n-8)x^{a+b} + 4mx^{a+c} + 4mx^{a+d} + 2(n-5)x^{2b} + 4x^{b+c} + 4x^{c+d} + 4x^{2d}; 2S_xJd_NM(\chi;x,y)|_{x=1}$$
$$= \frac{2m}{a} + \frac{2m(2n-8)}{a+b} + \frac{8m}{a+c} + \frac{8m}{a+d} + \frac{2(n-5)}{b} + \frac{8}{b+c} + \frac{4}{c+d} + \frac{4}{d}$$
$$d_NH(\chi) = \frac{1}{u}\{2mbd(a+b)(a+c)(a+d)(b+c)(c+d) + 2m(2n-8)abd(a+c)(a+d)(b+c)(c+d) + 8mabd(a+b)(a+d)(b+c)(c+d) + 8mabd(a+b)(a+c)(b+c)(c+d) + 4(n-5)ad(a+c)(a+d)(b+c)(c+d) + 8abd(a+b)(a+c)(a+d)(c+d) + 8abd(a+b)(a+c)(a+d)(b+c) + 4ab(a+b)(a+c)(a+d)(b+c)(c+d)\}$$
$$= m(2566822080 + 3833074924n + 2932551792n^2 + 1275066580n^3 + 318570084n^4 + 36104968n^5 + 3643164n^6 + 113288n^7) + m^2(5021045080 + 7920335216n + 6229404914n^2 + 2652419658n^3 + 615163682n^4 + 75460638n^5 + 4981764n^6 + 182000n^7 + 4704n^8) + m^3(5419378910 + 9213534072n + 6447240428n^2 + 2563585848n^3 + 519389002n^4 + 58160908n^5 + 4643528n^6 + 112448n^7) + m^4(3476853440 + 6446647114n + 4638363756n^2 + 1405778888n^3 + 256790698n^4 + 24650392n^5 + 1065816n^6) + m^5(1330973310 + 2712052284n + 1590787118n^2 + 475880176n^3 + 76067132n^4 + 4942744n^5) + m^6(288096900 + 651940590n + 349071702n^2 + 98334200n^3 + 12439288n^4) + m^7(30381000 + 78643800n + 45175700n^2 + 9883800n^3) + m^8(990000 + 3315000n + 3178400n^2) + 560893680 + 816357240n + 507891960n^2 + 144445800n^3 + 15027000n^4$$
$$where, u = abd(a+b)(a+c)(a+d)(b+c)(c+d)$$
$$= m(1856236320 + 3278958924n + 227227212n^2 + 998611036n^3 + 272379088n^4 + 49635936n^5 + 5588408n^6 + 193214n^7 + 8232n^8) + m^2(2928917040 + 5741936458n + 6891432945n^2 + 2139768366n^3 + 565462815n^4 + 84497222n^5 + 6458550n^6 + 337554n^7) + m^3(2508638680 + 5438153581 + 4654418900n^2 + 1998937210n^3 + 444106995n^4 + 47210194n^5 + 1842960n^6) + m^4(1259155880 + 3026609932n + 2524685270n^2 + 931658030n^3 + 151960130n^4 + 8649048n^5) + m^5(370375600 + 1001661115n + 758770835n^2 + 211190210n^3 + 18895620n^4) + m^6(59148000 + 188054850n + 116686500n^2 + 18363900n^3) + m^7(3960000 + 17320500n + 6991500n^2) + 495000m^9n + 492972480 + 778223160n + 442084500n^2 + 108108000n^3 + 9652500n^4$$

Theorem 9 *In Indu-Bala graph* $C_m \blacktriangledown C_n$*,where* $\forall m,n \geq 3$ *.Then d-Neighborhood Inverse Sum index* $d_N I(\chi)$ *is ;*

$$d_N I(\chi) = \frac{1}{v}(m(1260512076 + 7024493374n + 8318653512n^2 + 4901595930n^3 + 1866543936n^4 + 498840930n^5 + 89588214n^6 + 9876178n^7 + 602126n^8 + 15484n^9) + m^2(-608297850 + 9272536954n + 15314868054n^2 + 11113290658n^3 + 4586616021n^4 + 1125305691n^5 + 160730105n^6 + 12307381n^7 + 386246n^8) + m^3(-4235771482 + 3660897950n + 12728189788n^2 + 10757549147n^3 + 4325220792n^4 + 897684031n^5 + 92026448n^6 + 3698018n^7) + m^4(-5272109042 + 2052236254n + 4471905847n^2 + 4995479318n^3 + 1859352727n^4 + 234414721n^5 + 16694964n^6) + m^5(-3202006080 - 3657898711n + 72512094n^2 + 1073191293n^3 + 348596548n^4 + 92379583n^5) + m^6(-1053596750 - 1606381845n - 307119230n^2 + 93982575n^3 + 21825850n^4) + m^7(-179469000 - 325309900n - 43885550n^2 + 4835150n^3) + m^8(-12375000 - 25465500n + 1885500n^2) + 590268596 + 1870172960n + 1585180800n^2 + 520204000n^3 + 59040000n^4)$$

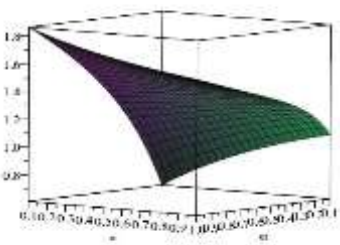
$$also, v = m(109479944 + 216502254n + 172400642n^2 + 74410088n^3 + 19324090n^4 + 3015166n^5 + 252448n^6 + 8232n^7) + m^2(149032086 + 320974418n + 279518381n^2 + 126776250n^3 + 30761761n^4 + 3621772n^5 + 154644n^6) + m^3(105593712 + 248441625n + 218262905n^2 + 89220090n^3 + 16541764n^4 + 1069740n^5) + m^4(41115970 + 107285975n + 87760985n^2 + 27785860n^3 + 2887848n^4) + m^5(8353800 + 25285050n + 17246880n^2 + 3136380n^3) + m^6(693000 + 2853900n + 1279500n^2) + m^7(99000n) + 32672640 + 59127200n + 38500000n^2 + 10780000n^3 + 1100000n^4$$

Proof 9 *We use the value of d-neighborhood polynomial and calculate the value of d-Neighborhood Inverse Sum index is*

$$d_N M(\chi; x, y) = 2mx^a y^a + m(2n-8)x^a y^b + 4mx^a y^c + 4mx^a y^d + 2(n-5)x^b y^b + 4x^b y^c + 4x^c y^d + 4x^d y^d; \; d_N I(\chi) = S_x J D_x D_y d_N M(\chi; x, y) at x = 1; \; D_x D_y d_N M(\chi, x, y)$$
$$= 2ma^2 x^{a-1} y^{a-1} + m(2n-8)ab x^{a-1} y^{b-1} + 4mac x^{a-1} y^{c-1} + 4mad x^{a-1} y^{d-1} + 2(n-5)b^2 x^{b-1} y^{b-1} + 4bc x^{b-1} y^{c-1} + 4cd x^{c-1} y^{d-1} + 4d^2 x^{d-1} y^{d-1}.; J D_x D_y d_N M(\chi, x, y)$$
$$= 2ma^2 x^{2a-2} + m(2n-8)ab x^{a+b-2} + 4mac x^{a+c-2} + 4mad x^{a+d-2} + 2(n-5)b^2 x^{2b-2} + 4bc x^{b+c-2} + 4cd x^{c+d-2} + 4d^2 x^{2d-2}; \quad S_x J D_x D_y d_N M(\chi, x, y)|_{x=1}$$
$$= \frac{m}{a-1}a^2 + \frac{2mn-8m}{a+b-2}ab + \frac{4m}{a+c-2}ac + \frac{4mad}{a+d-2} + \frac{n-5}{a-1}b^2$$
$$= \frac{4}{b+c-2}bc + \frac{4}{c+d-2}cd + \frac{2}{d-1}d^2$$
$$I(\chi) = \frac{1}{v}(ma^2(a+b-2)(a+c-2)(a+d-2)(b+c-2)(c+d-2)(d-1) + (2mn-8m)ab(a-1)(a+c-2)(a+d-2)(b+c-2)(c+d-2)(d-1) + 4mac(a-1)(a+b-2)(a+d-2)(b+c-2)(c+d-2)(d-1) + 4mad(a-1)(a+b-2)(a+c-2)(b+c-2)(c+d-2)(d-1) + (n-5)b^2(a+b-2)(a+c-2)(a+d-2)(b+c-2)(c+d-2)(d-1) + 4bc(a-1)(a+b-2)(a+c-2)(a+d-2)(c+d-2)(d-1) + 4cd(a-1)(a+b-2)(a+c-2)(b+c-2)(a+d-2)(d-1) + 2d^2(a-1)(a+b-2)(a+c-2)(a+d-2)(b+c-2)(c+d-2))$$
$$where, v = (a-1)(d-1)(a+b-2)(a+c-2)(a+d-2)(b+c-2)(c+d-2)$$
$$I(\chi) = \frac{1}{v}(m(1260512076 + 7024493374n + 8318653512n^2 + 4901595930n^3 + 1866543936n^4 + 498840930n^5 + 89588214n^6 + 9876178n^7$$

$$+602126n^8 + 15484n^9) + m^2(-608297850 + 9272536954n$$
$$+15314868054n^2 + 11113290658n^3 + 4586616021n^4 + 1125305691n^5$$
$$+160730105n^6 + 12307381n^7 + 386246n^8) + m^3(-4235771482$$
$$+3660897950n + 12728189788n^2 + 10757549147n^3 + 4325220792n^4$$
$$+897684031n^5 + 92026448n^6 + 3698018n^7) + m^4(-5272109042 + 2052236254n$$
$$+4471905847n^2 + 4995479318n^3 + 1859352727n^4 + 234414721n^5$$
$$+16694964n^6) + m^5(-3202006080 - 3657898711n + 72512094n^2 + 1073191293n^3$$
$$+348596548n^4 + 92379583n^5) + m^6(-1053596750 - 1606381845n - 307119230n^2$$
$$+93982575n^3 + 21825850n^4) + m^7(-179469000 - 325309900n - 43885550n^2$$
$$+4835150n^3) + m^8(-12375000 - 25465500n + 1885500n^2) + 590268596$$
$$+1870172960n + 1585180800n^2 + 520204000n^3 + 59040000n^4)$$
$$also, v = m(109479944 + 216502254n + 172400642n^2 + 74410088n^3 + 19324090n^4$$
$$+3015166n^5 + 252448n^6 + 8232n^7) + m^2(149032086 + 320974418n + 279518381n^2$$
$$+126776250n^3 + 30761761n^4 + 3621772n^5 + 154644n^6) + m^3(105593712$$
$$+248441625n + 218262905n^2 + 89220090n^3 + 16541764n^4 + 1069740n^5)$$
$$+m^4(41115970 + 107285975n + 87760985n^2 + 27785860n^3 + 2887848n^4)$$
$$+m^5(8353800 + 25285050n + 17246880n^2 + 3136380n^3) + m^6(693000$$
$$+2853900n + 1279500n^2) + m^7(99000n) + 32672640 + 59127200n + 38500000n^2$$
$$+10780000n^3 + 1100000n^4$$

(a)



(b)

Figure 5: (a) d-Neighborhood Harmonic Index for all m, n ; (b) d-Neighborhood Inverse sum Index for all m, n

4 Graphical Results and Their Discussion

A graph has wonderful information of the compound. Using the graphs we can describe the information that might not be expressed in terms of words. Here we have also shown the behavior of d-Neighborhood Polynomial and topological invariants by drawing graphics of Indu-Bala product of two cycle paths when one path is greater or equal and less than another path and vice versa. We have used different values of m and n and draw their respective graphs as shown in above Figure (3a) Third version of d-Neighborhood Zagreb index for all m, n. Figure (3b) d-Nhbd Second Zagreb Index for all m, n, Figure (3c) d-Nhbd forgotten Topological Index for all m, n, Figure (4a)d-Nieghbrhood General Randic Index for all m, n and $\alpha = 1$, Figure (4b)d-Neighbuhood third ND_e Index for all m, n, Figure (4c) d-Neighbuhood Fifth ND_e Index for all m, n,Figure (5a)d-Neighborhood Harmonic Index for all m, n and Figure (5b)d-Neighborhood Inverse sum Index for all m, n.

5 Conclusion

The research shows asizable calculation of some degree based indices of topology of Indu-Bala product of two cycles such as Third version of d-Neighborhood Zagreb index, d-Neighborhood second Zagreb index, d-Neighborhood forgotten topological index, d-Neighborhood general Randic index, d-Neighborhood Third NDe index, d-Neighborhood [1ex] Fifth NDe index, d-Neighborhood Harmonic index, d-Neighborhood inverse sum index. The standard polynomial which are linked with indicies of topology and are very much in vogue these days were also analyzed hectically for obtaining indu-bala product of two straits.We can also merged these results in QSAR for the proper inculcation of these analyses in the field of chemistry and medicines. It is very challenging to obtain new operators to derive all the degree based topological indices from d-Neighborhood polynomial.

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