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MATHEMATICAL MODELING AND SIMULATION OF SOCIETAL ABUSE OF WOMEN BASED ON SUPPORT SYSTEM INTERVENTIONS

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Abstract
Violence against women remains a critical societal challenge rooted in cultural norms, domestic pressures, poverty, limited education, weak legal structures, and Patriarchal society. This study develops a mathematical model to investigate the combined influence of Abused Women, Reported Women, Unreported Women, Support System, and Empowered Women on the dynamics of violence. The model demonstrates strong agreement with previously published data, underscoring the pivotal role of the Support System in reducing the violence population. While certain variables show strong initial effects, their long-term impact varies when examined across broader model conditions. The findings indicate that optimizing population and reporting data can substantially decrease the violence population and strengthen women's empowerment.

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INTRODUCTION

Women from all around the world face challenges like violence, abuse, discrimination, limited access to resources, and unequal opportunities in many sectors such as education, work and health etc [1]. These issues, combined with socio-economic differences, cultural traditions, and violence against women, impact their physical and mental health, especially are those countries that are underdeveloped, which in turn disrupts the development and stability of societies. Further than the immediate harm to women, these challenges also continue cycles of poverty, inequality, and social instability.

Cultural and legal barriers in male-dominated traditions and weak laws in many societies increase these problems [10]. For example, Patriarchal traditions often prevent women from reporting violence and getting justice or seeking help, while weak legal frameworks allow offenders to escape punishment [15, 16]. Additionally, insufficient mental health resources further deprive affected women of the support they need to recover. According to global statistics, including WHO reports, one in three women experiences physical or sexual violence, emphasizing the critical need for immediate and effective interventions and Reports from UN Women and the Aurat Foundation indicate that nearly 90% of Pakistani women experience some

form of emotional, physical, or psychological abuse [2,18].

Research on violence against women has provided important information on the causes of the prevalence of this problem and ways to address it [3, 14].Violence against women is not only a violation of human rights but also a serious public health crisis that has profound effects on the affected individuals and society as a whole [4, 5]. A mathematical framework for modeling provided valuable insights into the dynamics of abuse; it did not explicitly include the role of support systems in reducing violence. This omission is important because growing evidence suggests that structured interventions such as counseling, legal assistance, and community support play a significant role in empowering women and reducing rates of abuse [6, 7].

Mathematical models have emerged as powerful tools for systematically analyzing these challenges and proposing interventions to address them [11]. These models provide deep insights into the dynamics of violence reporting behaviors and recovery processes. However, existing models often distort real-world difficulties because they do not include the structured support systems that are important in ending violence [8]. The belief system includes a wide range of interventions, including helplines, legal aid and counseling services, awareness programs, and empowerment programs, which are critical in breaking the cycle of violence [9, 10].

Building on previous research, this study introduces a new section to the existing mathematical model that represents support systems. The purpose of including this compartment is to provide resources, counseling, and legal assistance to affected and abused women [16]. By adding this new component, the model will analyze the role of institutional and community-based support to determine the extent to which it helps reduce violence and empower women [16, 17]. Furthermore, this study aims to explore the relationship between support systems and other important factors, such as reporting rates, public awareness, and social attitudes, to develop effective approaches to addressing violence against women [7].

Associating the gap between theoretical frameworks and real-world interventions, this research provides a comprehensive perspective on understanding the effective role of support systems that can help reduce violence and promote gender equality [9, 12]. The study highlights the necessity for a multifaceted strategy to address this problem, combining mathematical modeling, policy analysis, and community engagement to address one of society's most pressing challenges.

The paper proceeds by introducing the mathematical framework that captures the transitions among the different groups in the system. This is followed by an examination of the model's positivity and boundedness to confirm its biological and mathematical validity. We continue with an analysis of the system's stability under various conditions. The closing section offers a summary of the results and suggests potential avenues for further investigation.

Mathematical Model

In this section, we present an extended mathematical model to investigate the population dynamics associated with violence against women (VAW) [6]. This model builds upon a nonlinear system of ODEs, with essential modifications suited to our context [13, 14]. Specifically, we incorporate a new compartment: the Support System $S(t)$, which plays a critical role in mitigating violence through mechanisms such as Counseling $C(t)$ and Empowerment $E(t)$.

In Figure 1 each compartment interacts within a nonlinear framework to capture the transitions from one state to another. For instance, abused women may either report the violence and transition to the reported class $R(t)$, or remain unreported in $U(t)$. With the help of support system interventions, including counseling and social support, women may transition into $C(t)$ and eventually into the empowered class $E(t)$, thus reducing their future vulnerability to abuse.

By incorporating $S(t)$ and modifying the structure of the original system, this model provides a more comprehensive representation of

real-world social dynamics. It enables the assessment of the effectiveness of intervention policies aimed at reducing violence and supporting survivors of abuse.

The model is represented as:

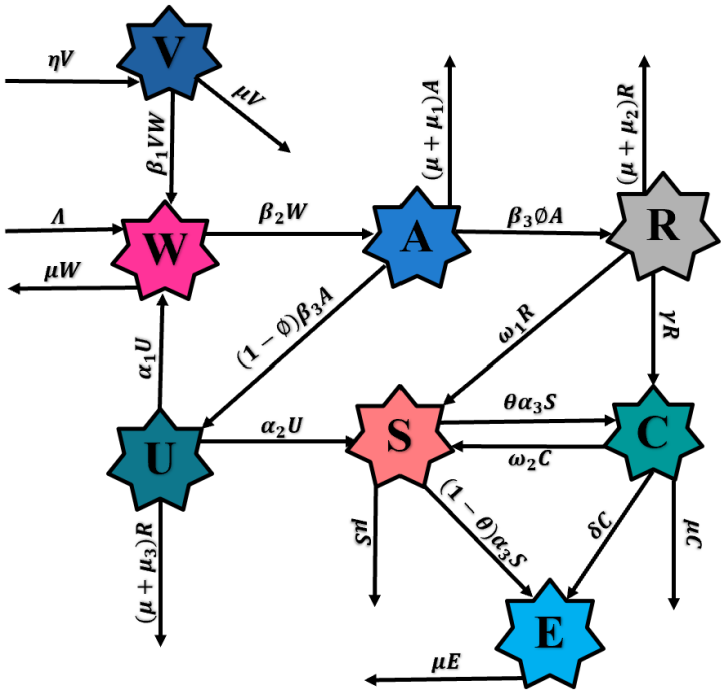


Figure 1. **Flow diagram of the model structure.**

This diagram shows how individuals move between the different groups, including violence, vulnerability, abuse, reporting, support, counseling, and empowerment. Each arrow indicates the direction of movement, and the labels show the factors influencing these transitions.

$$\frac{dV}{dt} = \eta V - \beta_1 VW - \mu V \quad (1)$$

$$\frac{dW}{dt} = \Lambda + \beta_1 VW + \alpha_1 U - \beta_2 W - \mu W \quad (2)$$

$$\frac{dA}{dt} = \beta_2 W - \beta_3 \phi A - (1-\phi)\beta_3 A - (\mu + \mu_1)A \quad (3)$$

$$\frac{dR}{dt} = \beta_3 \phi A - \gamma R - \mu R - \omega_1 R \quad (4)$$

$$\frac{dU}{dt} = (1-\phi)\beta_3 A - \alpha_1 U - \mu U - \omega_2 U \quad (5)$$

$$\frac{dS}{dt} = \omega_1 R + \omega_2 U - \alpha_2 S - \mu S \quad (6)$$

$$\frac{dC}{dt} = \alpha_2 S - \theta C - \mu C \quad (7)$$

$$\frac{dE}{dt} = \theta C - \mu_2 E \quad (8)$$

Use Label Notations

$$\begin{aligned} k_1 &= \beta_2 + \mu, & k_2 &= \beta_3 + \mu + \mu_1, \\ k_3 &= \gamma + \mu + \mu_2 + \omega_2, & k_4 &= \alpha_1 + \mu_3 + \mu, \\ k_5 &= \alpha_3 + \mu, & k_6 &= \delta + \omega_2 + \mu. \end{aligned}$$

The system can be rewritten using labeled notations as follow:

$$\begin{aligned} \frac{dV}{dt} &= \eta V - \beta_1 VW - \mu V, \\ \frac{dW}{dt} &= \Lambda + \beta_1 VW + \alpha_1 U - k_1 W, \\ \frac{dA}{dt} &= \beta_2 W - k_2 A, \\ \frac{dR}{dt} &= \beta_3 \phi A - k_3 R, \\ \frac{dU}{dt} &= (1-\phi)\beta_3 A - k_4 U, \\ \frac{dS}{dt} &= \alpha_2 U + \omega_1 R + \omega_2 C - k_5 S, \\ \frac{dC}{dt} &= \gamma R + \theta \alpha_3 S - k_6 C, \\ \frac{dE}{dt} &= (1-\theta)\alpha_3 S + \delta C - \mu E. \end{aligned} \quad (9)$$

Table 1: Summary of Model Parameters

Parameter	*Description*
η	Influence rate of violence.
Λ	Recruitment rate of women population.
β_1	Interaction rate between violent behavior and susceptible women.
β_2	Rate of susceptible women moving into abuse.
β_3	Transmission rate from abused to reported/unreported population.
μ	Natural death/removal rate (applies to multiple compartments).
μ_1	Transition rate of death due to abuse.
μ_2	Rate of deaths after reporting violence.
μ_3	Rate of suicide or honor killings.
α_1	Rate of unreported women returning to susceptible.
α_2	Rate of unreported women entering the support system.
α_3	Rate of support class accessing counseling.
ω_1	Transmission rate from reported to support system.

ω_2	Rate of individuals returning from counseling to support.
γ	Recovery rate from reported population.
ϕ	Fraction corresponding to movement from abuse to report.
$1-\phi$	Fraction corresponding to movement from abuse to unreport.
δ	Rate of counseling women becoming empowered.
θ	Fraction of support class receiving counseling.
$1-\theta$	Proportion remaining in support system but not counseled.

Positivity and Boundedness of the Model

Theorem: (Positivity) Consider the system of ordinary differential equations

$$\frac{dX}{dt} = f(X), X(0) \geq 0$$

Where $X \in \mathbb{R}^n$ and $f: \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable. If, for each $i \in \{1,2,\dots,n\}$, $X_i = 0$ and $X_j \geq 0 (j \neq i) \Rightarrow f_i(X) \geq 0$.

Then the non-negative orthant

$$\mathbb{R}^n_+ = \{X \in \mathbb{R}^n \mid X_i \geq 0, \forall i\}$$

is positively invariant under the flow of the system. That is, if $X(0) \in \mathbb{R}^n_+$, then $X(t) \in \mathbb{R}^n_+$ for all $t \geq 0$.

Positivity of the Model

Let all parameters be nonnegative with $0 \leq \phi, \theta \leq 1$, and let the initial conditions satisfy

$$V(0), W(0), A(0), R(0), U(0), S(0), C(0), E(0) \geq 0$$

Then the solution of the system satisfies

$$V(t), W(t), A(t), R(t), U(t), S(t), C(t), E(t) \geq 0 \text{ for all } t \geq 0$$

To verify the conditions of the theorem, we examine the vector field on each boundary of the nonnegative orthant, setting one variable to zero while keeping the others nonnegative:

Therefore, the region

$$\Omega = \{(V, W, A, R, U, S, C, E) \in \mathbb{R}^8 \geq 0\}$$

is positively invariant.

Furthermore, the right-hand side of the system is locally Lipschitz on Ω, ensuring existence, uniqueness, and forward extension of solutions. Thus, nonnegative initial conditions guarantee that all state variables remain nonnegative for all $t \geq 0$.

Theorem: (Boundedness) If $f: [a,b] \to \mathbb{R}$ is continuous on the closed, bounded interval $[a,b]$, then there exist real numbers m and M such that

$$m \leq f(x) \leq M \text{ for all } x \in [a,b].$$

And f attains both its minimum and maximum on $[a,b]$.

Boundedness of the Model

Define the total population in the system as:

$$N(t) = V(t) + W(t) + A(t) + R(t) + U(t) + S(t) + C(t) + E(t).$$

Adding all the differential equations yields:

$$\frac{dN}{dt} \leq \Lambda - \mu_{\min} N$$

Where

$$\mu_{\min} = \min(\mu, \mu_1, \mu_2, \mu_3, \delta) > 0$$

Solving this inequality gives:

$$N(t) \leq \frac{\Lambda}{\mu_{\min}} + \left(N(0) - \frac{\Lambda}{\mu_{\min}}\right) e^{-\mu_{\min} t}$$

Thus, $N(t)$ is bounded above by $\frac{\Lambda}{\mu_{\min}}$. This implies that the total population remains finite for all $t > 0$, and no population class grows without bound.

Stability Analysis

To initiate a qualitative analysis of system (9), we first determine its equilibrium points. Two principal steady states emerge: the violence-free equilibrium (characterizing the absence of violence) and the violence-present equilibrium (characterizing sustained violence). We then analyze existence criteria and local/global stability for each equilibrium to reveal the parameter

regimes that lead to eradication or persistence of violence.

Violence-Free Equilibrium Point

The violence-free equilibrium point (VFE) is given by:

$$\mathcal{E}^0 = (\mathrm{V}^0, \mathrm{W}^0, \mathrm{A}^0, \mathrm{R}^0, \mathrm{U}^0, \mathrm{S}^0, \mathrm{C}^0, \mathrm{E}^0)$$

where

$$\mathrm{V}^0 = 0, \mathrm{W}^0 = \frac{\Lambda k_2 k_4}{Z}, \mathrm{A}^0 = \frac{\Lambda \beta_2 k_4}{Z},$$
$$\mathrm{R}^0 = \frac{\Lambda \beta_2 \beta_3 k_4 \varphi}{k_3 Z}, \mathrm{U}^0 = \frac{\Lambda \beta_2 \beta_3 (1-\varphi)}{Z},$$
$$Z = k_1 k_2 k_4 - \alpha_1 (1-\varphi)\beta_3 \beta_2,$$
$$\mathrm{S}^0 = \frac{k_6 \alpha_2 U_0 + R_0 (k_6 \omega_1 + \omega_2 \gamma)}{k_6 k_5 - \omega_2 \theta \alpha_3},$$
$$\mathrm{C}^0 = \frac{\theta \alpha_3 \alpha_2 U_0 + R_0 (\theta \alpha_3 \omega_1 + k_5 \gamma)}{k_5 k_6 - \alpha \theta \alpha_3 \omega_2},$$
$$\mathrm{E}^0 = \frac{\delta \gamma R_0 + \alpha S_0}{k_6 (1-\theta) + \delta \theta} \frac{1}{\mu}.$$
$$\mathcal{E}^0 = (0, W^0, A^0, R^0, U^0, S^0, C^0, E^0)$$

The Jacobian matrix of the system (9) is given as:

$$\mathcal{J} = \begin{bmatrix} s_1 & -\beta_1 V & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 W & \mathrm{s}_2 & 0 & 0 & \alpha_1 & 0 & 0 & 0 \\ 0 & \beta_2 & \mathrm{s}_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 \phi & -\mathrm{s}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\phi)\beta_3 & 0 & -\mathrm{s}_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_1 & \alpha_2 & -\mathrm{s}_6 & \omega_2 & 0 \\ 0 & 0 & 0 & \gamma & 0 & \theta\alpha_3 & -\mathrm{s}_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\theta)\alpha_3 & \delta & -\mu \end{bmatrix}$$

Where $s_1 = \eta - \beta_1 W - \mu, s_2 = \beta_1 V - (\beta_2 + \mu), s_3 = -\beta_3 - \beta_3 \phi - \mu - \mu_1, s_4 = (\gamma + \mu + \mu_2 + \omega_2), s_5 = (\alpha_1 + \alpha_2 + \mu + \mu_3), s_6 = \alpha_3 - \mu, s_7 = (\delta + \omega_2 + \mu)$

To study the local stability of the violence-free equilibrium point $\mathcal{E}^0$, we substitute its values into the Jacobian matrix $\mathcal{J}$. The resulting form is:

$$\mathcal{J}(\mathcal{E}^0) = \begin{pmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 & M_{25} & 0 & 0 & 0 \\ 0 & M_{32} & M_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{43} & M_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{53} & 0 & M_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{64} & M_{65} & M_{66} & M_{67} & 0 \\ 0 & 0 & 0 & M_{74} & 0 & M_{76} & M_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{86} & M_{87} & M_{88} \end{pmatrix}$$

where the entries are:

$$M_{11} = -W^0 \beta_1 - \mu + \eta, M_{21} = W^0 \beta_1, M_{22} = -\beta_2 - \mu, M_{25} = \alpha_1$$
$$M_{32} = \beta_2, M_{33} = -\beta_3 - \mu - \mu_1, M_{43} = \beta_3 \phi, M_{44} = -\gamma - \mu - \mu_2 - \omega_2$$
$$M_{53} = \beta_3 (1-\phi), M_{55} = -\alpha_1 - \alpha_2 - \mu - \mu_3, M_{64} = \omega_1, M_{65} = \alpha_2$$
$$M_{66} = -\alpha_3 - \mu, M_{67} = \omega_2, M_{74} = \gamma, M_{76} = \theta \alpha_3, M_{77} = -\delta - \omega_2 - \mu$$
$$M_{86} = (1-\theta)\alpha_3, M_{87} = \delta, M_{88} = -\mu$$

Here, $W^0 = \frac{\Lambda k_2 k_4}{Z}$, with $Z = k_1 k_2 k_4 - \alpha_1(1-\phi)\beta_3\beta_2$.

Gershgorin Circle Theorem: Let $J = [a_{ij}]$ be a real square matrix. If, for every row i, the diagonal entry is more negative than the sum of the absolute values of all the other entries in that row, that is,

$$a_{ii} < -\sum_{j \neq i} |a_{ij}|.$$

Then all eigenvalues of J have negative real parts. Hence, the equilibrium of the system is locally asymptotically stable.

The latent values of the matrix $\mathcal{J}(\mathcal{E}^0)$ are $W^0\beta_1 + \mu > \eta, -\mu$, the remaining $\mathcal{J}(\mathcal{E}^0)$ evaluated at the violence-free equilibrium (i.e., $V = 0$) by using the Gershgorin Circle Theorem [16]. Applying Gershgorin's Circle Theorem, the following conditions are obtained for local asymptotic stability of $\mathcal{J}(\mathcal{E}^0)$:

Row	Diagonal Entry	Off-Diagonal Entries	Gershgorin Condition
1	$-\beta_3 - \mu - \mu_1$	β_2	$\beta_3 + \mu + \mu_1 > \beta_2$
2	$-\gamma - \mu - \mu_2 - \omega_2$	$\beta_3\phi$	$\gamma + \mu + \mu_2 + \omega_2 > \beta_3\phi$
3	$-\alpha_1 - \alpha_2 - \mu - \mu_3$	$\beta_3(1-\phi)$	$\alpha_1 + \alpha_2 + \mu + \mu_3 > \beta_3(1-\phi)$
4	$-\alpha_3 - \mu$	$\omega_1 + \alpha_2 + \omega_2$	$\alpha_3 + \mu > \omega_1 + \alpha_2 + \omega_2$
5	$-\delta - \omega_2 - \mu$	$\gamma + \theta\alpha_3$	$\delta + \omega_2 + \mu > \gamma + \theta\alpha_3$

If all inequalities are satisfied, the violence-free equilibrium is locally asymptotically stable.

Violence-Present Equilibrium Point

The violence-present equilibrium point (VPE) is given by:

$$\mathcal{E}^* = (V^*, W^*, A^*, R^*, U^*, S^*, C^*, E^*)$$

where

$$V^* = \frac{-\Lambda\beta_1 k_2 k_4 - \alpha[\beta_2\beta_3(1-\varphi)(\eta-\mu) + (\eta-\mu)k_1 k_2 k_4]}{\beta_1 k_2 k_4(\eta-\mu)},$$

$$W^* = \frac{\eta-\mu}{\beta_1}, A^* = \frac{\beta_2(\eta-\mu)}{k_2\beta_1},$$

$$R^* = \frac{\beta_2\beta_3\varphi(\eta-\mu)}{\beta_1 k_2 k_3}, U^* = \frac{\beta_2\beta_3(1-\varphi)(\eta-\mu)}{\beta_1 k_2 k_4},$$

$$S^* = \frac{k_6\alpha_2 U^* + R^*(k_6\omega_1 + \omega_2\gamma)}{k_6 k_5 - \omega_2\theta\alpha_3},$$

$$C^* = \frac{\theta\alpha_3\alpha_2 U^* + R^*(\theta\alpha_3\omega_1 + k_5\gamma)}{k_5 k_6 - \alpha\theta\alpha_3\omega_2},$$

$$E^* = \frac{\delta\gamma R^* + \alpha S^*[k_6(1-\theta) + \delta\theta]}{k_6\mu}.$$

$$\mathcal{E}^* = (V^*, W^*, A^*, R^*, U^*, S^*, C^*, E^*)$$

At the violence-present equilibrium $\mathcal{E}^*$, the Jacobian becomes:

$$\mathcal{J}(\mathcal{E}^*) = \begin{pmatrix} 0 & N_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ N_{21} & N_{22} & 0 & 0 & N_{25} & 0 & 0 & 0 \\ 0 & N_{32} & N_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{43} & N_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{53} & 0 & N_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{64} & N_{65} & N_{66} & N_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{76} & N_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{86} & N_{87} & N_{88} \end{pmatrix}$$

with entries:

$$N_{12} = -W^*\beta_1, N_{21} = W^*\beta_1, N_{22} = -\beta_2 - \mu, N_{25} = \alpha_1,$$
$$N_{32} = \beta_2, N_{33} = -\beta_3 - \mu - \mu_1, N_{43} = \beta_3\phi, N_{44} = -\gamma - \mu - \mu_2 - \omega_2,$$
$$N_{53} = \beta_3(1-\phi), N_{55} = -\alpha_1 - \alpha_2 - \mu - \mu_3, N_{64} = \omega_1, N_{65} = \alpha_2,$$
$$N_{66} = -\alpha_3 - \mu, N_{67} = \omega_2, N_{76} = \theta\alpha_3, N_{77} = -\delta - \omega_2 - \mu,$$
$$N_{86} = (1-\theta)\alpha_3, N_{87} = \delta, N_{88} = -\mu.$$

Here; $W^* = \frac{\eta-\mu}{\beta_1}, V^* = \frac{-\Lambda\beta_1 k_2 k_4 - \alpha_1[\beta_2\beta_3(1-\phi)(\eta-\mu)+(\eta-\mu)k_1k_2k_4]}{\beta_1 k_2 k_4(\eta-\mu)}$

The latent values of the matrix $J(\varepsilon^*)$ are $-\mu$, the remaining $\mathcal{J}(\mathcal{E}^*)$ by using the Gershgorin Circle Theorem. Applying Gershgorin's theorem at $\mathcal{J}(\mathcal{E}^*)$, we obtain the following conditions:

Row	Diagonal Entry	Off-Diagonal Entries	Gershgorin Condition
1	0	$\eta - \mu$	$0 > \eta - \mu$
2	$-\beta_2 - \mu$	$\eta - \mu + \alpha_1$	$\beta_2 + \mu > \eta - \mu + \alpha_1$
3	$-\beta_3 - \mu - \mu_1$	β_2	$\beta_3 + \mu + \mu_1 > \beta_2$
4	$-\gamma - \mu - \mu_2 - \omega_2$	$\beta_3\phi$	$\gamma + \mu + \mu_2 + \omega_2 > \beta_3\phi$
5	$-\alpha_1 - \alpha_2 - \mu - \mu_3$	$\beta_3(1-\phi)$	$\alpha_1 + \alpha_2 + \mu + \mu_3 > \beta_3(1-\phi)$
6	$-\alpha_3 - \mu$	$\omega_1 + \alpha_2 + \omega_2$	$\alpha_3 + \mu > \omega_1 + \alpha_2 + \omega_2$
7	$-\delta - \omega_2 - \mu$	$\theta\alpha_3$	$\delta + \omega_2 + \mu > \theta\alpha_3$

If these inequalities conditions hold, the violence-present equilibrium is locally asymptotically stable.

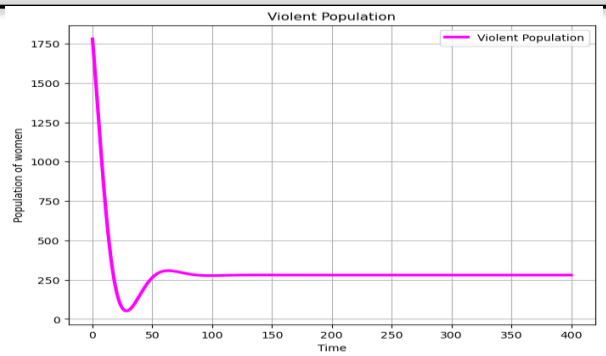


Figure 2: Change in the number of individuals showing violent behavior over time.

There is an early decline followed by a gradual rise to a stable value. This pattern reflects the balance between the factors that promote harmful actions and those that limit or reduce them.

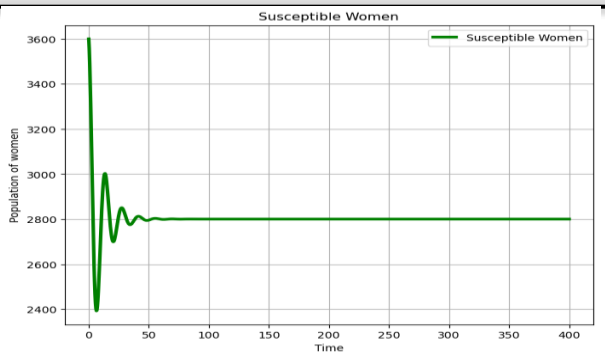


Figure 3: Population of women who are vulnerable to abuse.

The curve shows fluctuations at the beginning due to interactions with harmful and protective groups. Over time, the population becomes stable, indicating the combined influence of awareness, prevention programs, and social responses.

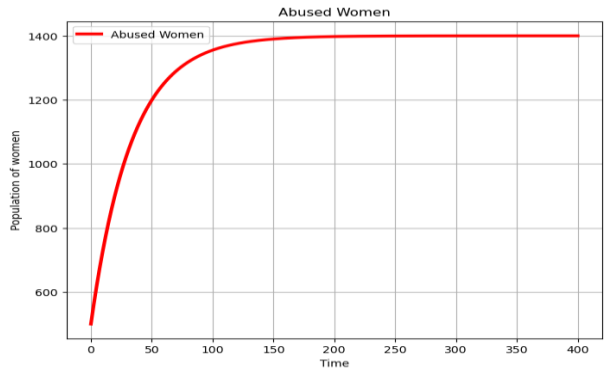


Figure 4: Number of women experiencing abuse over time.

The abused population rises before reaching a stable level. This trend shows how the burden of abuse can remain high when preventive and protective measures are not strong enough.

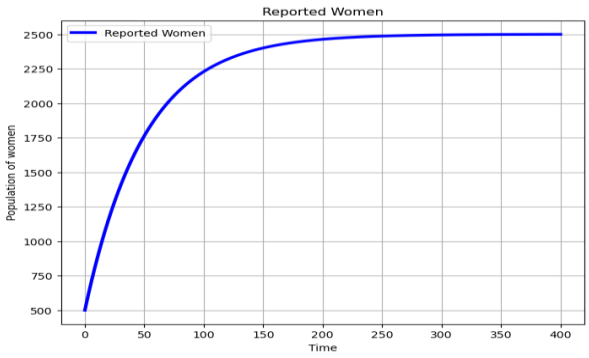


Figure 5: Reported cases of abuse.

The reported cases increase steadily and then level off. This reflects the impact of awareness, access to reporting channels, and the confidence women gain when support services are available.

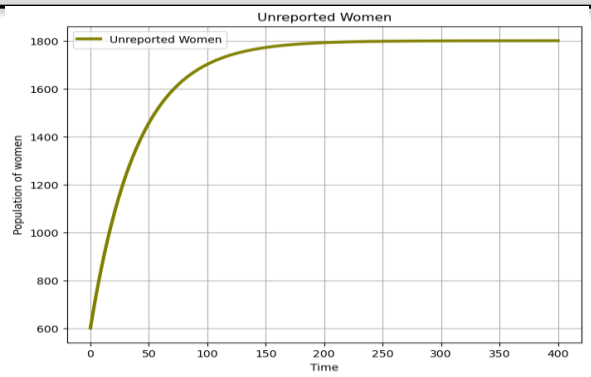


Figure 6: Unreported cases of abuse.
The number of unreported cases grows rapidly and remains high. This illustrates the ongoing challenges that prevent many women from seeking help, such as fear, stigma, or limited access to support.

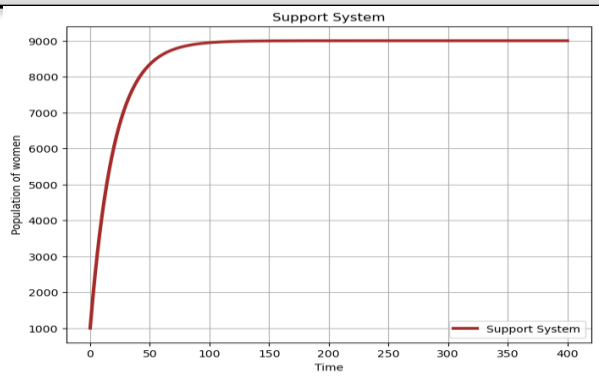


Figure 7: Growth of the support system population.
As more cases come into contact with formal or informal help sources, the support system expands. The pattern highlights the increasing need for services such as legal help, safe spaces, and community assistance.

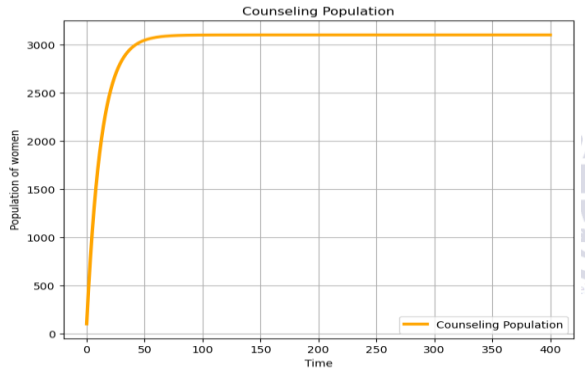


Figure 8: Women receiving counseling services.
Counseling numbers rise quickly and then stabilize. This group represents the first stage of structured help, where affected individuals begin receiving guidance and emotional support.

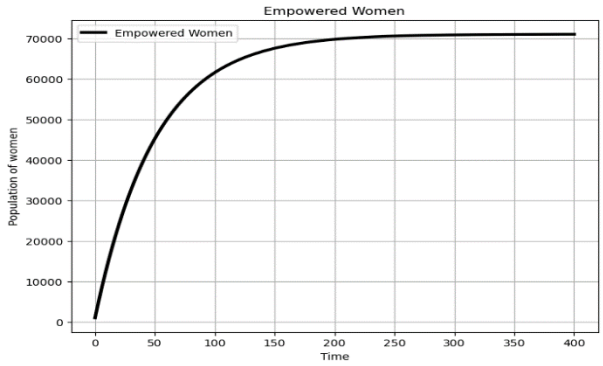


Figure 9: Population of empowered women.
The empowered group grows steadily and eventually becomes the largest among all compartments. This shows the long-term effect of effective support and counseling, helping women regain strength and reduce future vulnerability.

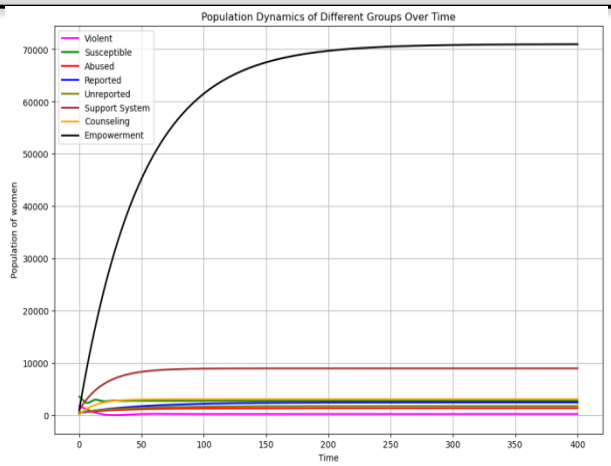


Figure 10: Full system behavior showing the interaction of all groups.

The figure presents the overall development of every compartment in the model. It demonstrates how violence, reporting, support, counseling, and empowerment evolve together and shape the long-term outcome for the population.

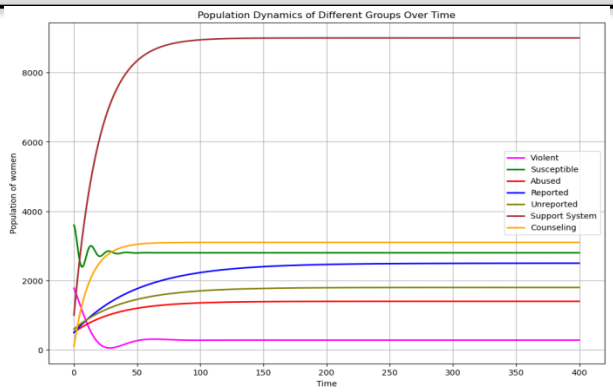


Figure 11: System behavior without the empowerment group for clearer visualization.

Removing the empowerment curve allows the smaller groups to be viewed more clearly. This helps highlight patterns in abused, reported, unreported, support, and counseling populations that are otherwise overshadowed by the large empowerment values.

Conclusion

The dynamics of women's abuse using a mathematical model that highlights the crucial role of support systems, represented as $S(t)$. The findings show that when support is weak, cycles of violence intensify and women remain trapped in unsafe environments with limited access to justice and care. However, when $S(t)$ is strengthened through family support, legal protection, counseling, and community awareness, the system moves toward a Violence-Free Equilibrium, where safety and stability become possible. This approach demonstrates how mathematical tools can effectively explain social problems and predict how interventions can change long-term outcomes.

The study also bridges theory and practice by revealing how improvements in support networks can reduce abuse at the societal level. Although the model simplifies a complex issue and cannot capture every real-life factor, it provides a strong foundation for future research and policymaking. Overall, the work shows that strengthening is essential for reducing abuse and creating a safer society and it encourages further interdisciplinary collaboration to build more accurate and impactful models.

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